Overview of Open-Loop Frequency Estimation Techniques for Grid Synchronization of Single-Phase Power Converters

Hachemi Rahmani, Boulouma Sabri, and Mourad Haddadi

Abstract—Open-loop synchronization techniques allow to overcome the stability issues to which closed-loop techniques are subject. Most open-loop techniques differ in their Quadratic Signal Generator and their frequency estimation method. In this paper, an overview of the main frequency estimation methods for single-phase power systems is presented. A description of each technique with their structure for numerical implementation is carried out. A comparative study of those frequency estimation methods is performed according to three test scenarios in order to evaluate the robustness of each method. Recommendations for the selection of the technique is given according to the simulation results.

Keywords—Open-loop grid synchronization, Frequency estimation, grid-connected power inverter.

I. INTRODUCTION

The current concept of centralized and unidirectional energy flow utility grids is unsustainable for economic, security, reliability, and ecological reasons [1]. The integration of decentralized energy resources addresses these issues, particularly with renewable energy sources like photovoltaic and wind. However, to ensure good energy quality, these resources must inject current according to established standards. Ideally, the grid voltage waveform is a pure sine wave with fixed amplitude and frequency at nominal values. In reality, they are subject to fluctuations due to grid conditions. For instance, voltage sags along with phase jumps can occur during the startup of powerful motors [2], while frequency shifts happen during load commutations [3].

In practice, ZCD can be realized by an electronics board, like a Digital Signal Processor (DSP). Considering the grid voltage \( v(t) \) given by:

\[
v(t) = V_m \cos(\theta(t)) = V_m \cos(2\pi ft + \phi)
\]

Were \( V_m \) is the amplitude, \( \theta \) is the phase angle, \( f \) is the frequency and \( \phi \) is the initial phase angle.

In-phase signal \( v_\alpha \) and quadratic signal \( v_\beta \) are expressed by:

\[
\begin{align*}
v_\alpha(t) &= V_m \cos(\theta(t)) \\
v_\beta(t) &= V_m \sin(\theta(t))
\end{align*}
\]

Given this two signals, amplitude \( V_m \) and phase angle \( \theta \) are obtained at instant \( t \) by:

\[
V_m = \sqrt{v_\alpha^2 + v_\beta^2}
\]

\[
\theta = \tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right)
\]

Although OL amplitude and phase estimations are somehow satisfying, they have a dynamic behavior limited by their stability conditions. OL techniques do not have this problem, as they are unconditionally stable. In recent years, modern OL synchronization techniques emerged, whose performance can compete those of closed-loop ones [5].

II. SYNCHRONIZATION BY ZERO-CROSSING DETECTION

A simple method to estimate grid frequency and phase angle is first, to detect instants where the AC grid voltage crosses zero, then evaluate the duration between two instants to measure the grid period. Phase angle is obtained by integrating frequency. In practice, ZCD can be realized by an electronics board, like a
half-wave rectifier with a voltage comparator that gives a square signal with a 50% duty cycle, toward a microcontroller. The rising and falling edges of this signal call a microcontroller interrupt subroutine that measure half a period of the input signal and calculate its frequency [6]. ZCD can also be achieved numerically, by observing sign changes of the input signal samples [7].

This method presents several disadvantages. When the ZCD is performed numerically, it requires a high sampling frequency in order to minimize estimated frequency errors. Moreover, this technique is too sensitive to grid disturbances. For example, multiple zero-crossing can be detected if the grid voltage is polluted by harmonics or frequency estimation can be incorrect if the signal input signal present a DC offset. To solve this issue, a sharp filter can be used to remove voltage distortions. The problem is that the sharper the filter, the slower is the dynamic response.

III. MODERN OPEN-LOOP FREQUENCY ESTIMATION TECHNIQUES

There are several OL frequency estimation techniques in the literature. Some of them require QSG, some do not. All those techniques can estimate input signal frequency using few samples. They can be classified by standard methods, consecutive samples methods, and Teager Energy Operator method.

A. Standard Methods

One of the most intuitive method to estimate the grid frequency is to differentiate its phase angle, using (4) [8]:

$$\dot{\omega} = \frac{d\hat{\theta}}{dt} = \frac{\dot{\hat{v}}_\alpha \dot{\hat{v}}_\beta - \dot{\hat{v}}_\beta \dot{\hat{v}}_\alpha}{\hat{v}_\alpha^2 + \hat{v}_\beta^2} \tag{5}$$

The hat over a variable mean its estimated value and the dot over a variable mean its derivative.

In discrete-time, this give the following differences equation:

$$\hat{\omega}_{\text{std}} = \frac{1}{T_s} \left( \frac{\hat{v}_\alpha(n-1)\hat{v}_\beta(n) - \hat{v}_\beta(n-1)\hat{v}_\alpha(n)}{\hat{v}_\alpha^2(n) + \hat{v}_\beta^2(n)} \right) \tag{6}$$

Where $T_s$ is the sampling period. This method presents a steady-state error that increase when the switching frequency decrease [9]. To correct this error, an enhanced version that introduce inverse sinus function, was developed [10].

$$\hat{\omega}_{\text{std}} = \frac{1}{T_s} \sin^{-1} \left( \frac{\hat{v}_\alpha(n-1)\hat{v}_\beta(n) - \hat{v}_\beta(n-1)\hat{v}_\alpha(n)}{\hat{v}_\alpha^2(n) + \hat{v}_\beta^2(n)} \right) \tag{7}$$

This lead to enhanced version of 3CS and 4CS (E3CS and E4CS), described by the following equations:

$$\hat{\omega}_{\text{E3CS}} = \frac{1}{T_s} \cos^{-1} \left( \frac{\hat{v}_\alpha(n)\hat{v}_\alpha(n-1) + \hat{v}_\beta(n)\hat{v}_\beta(n-1)}{\hat{v}_\alpha^2(n) + \hat{v}_\beta^2(n)} \right) \tag{8}$$

$$\hat{\omega}_{\text{E4CS}} = \frac{1}{T_s} \cos^{-1} \left( \frac{(\hat{v}(n) - \hat{v}(n-1)) + (\hat{v}(n) - \hat{v}(n-2))}{2(\hat{v}(n) - \hat{v}(n-2))} \right) \tag{9}$$

The major disadvantage of relying on a QSG is that the estimated frequency is subject to errors when $v_\alpha$ and $v_\beta$ are not perfectly in quadrature, which can happen for example when the QSG is sensitive to frequency deviation or in case of unbalanced three-phase system.
\[ \hat{\omega}_{E4CS} = \frac{1}{T_s} \cos^{-1} \left( \frac{(M_{1\alpha} + M_{1\beta}) + (M_{2\alpha} + M_{2\beta})}{2(M_{2\alpha} + M_{2\beta})} \right) \]  \hspace{1cm} (12) \\

With

\[ \begin{cases} M_{1x} = v_x(n)v_x(n-1) - v_x(n-1)v_x(n-3) \\ M_{2x} = v_x(n-1) - v_x(n-1)v_x(n-2) \end{cases} \] \hspace{1cm} (13) \\

Where \( x \) can be either \( \alpha \) or \( \beta \).

4CS and E4CS methods are the only ones to be immune to the presence of DC offset in the input signal \([12], [13]\\). \hspace{1cm} (12) \\

Moreover, if (14) is applied on the derivative of \( v(t) \), it gives :

\[ \psi[v(t)] = V_m^2\omega^4 \] \hspace{1cm} (16) \\

Combining (15) with (16), amplitude \( V_m \) and angular frequency \( \omega \) are obtained by the following formulas:

\[ \begin{cases} V_m = \frac{\psi[v(t)]]}{\sqrt{\psi[v(t)]]} \\ \omega = \sqrt{\psi[v(t)]} \end{cases} \] \hspace{1cm} (17) \\

In discrete time (14) becomes:

\[ \psi[x(n)] = \frac{(x^2(n) - x(n+1)x(n-1))}{T_s^2} \] \hspace{1cm} (18) \\

The estimated amplitude and frequency are then obtained by:

\[ \begin{cases} V_m = \frac{2\psi[v(n)]}{\sqrt{\psi[v(n+1)] - \psi[v(n-1)]}} \\ \omega = \sqrt{\frac{\psi[v(n+1)] - \psi[v(n-1)]}{4\psi[v(n)]}} \end{cases} \] \hspace{1cm} (19) \\

The major concern of the TEO is its high sensitivity to noises. Indeed, the use of derivative increases input signal noises and greatly corrupt the frequency estimation. For this reason, some works associate this method with a very sharp filter like the filter based on Recursive Discrete Fourier Transform (RDFT) and inverse RDFT (IRDFT) \([16]–[19]\\). In practice, using a RDFT-IRDFT filter brings other issues like rounding errors and instability especially when using its frequency adaptive version \([20]–[23]\\).

**C. Teager Energy Operator**

The Teager Energy Operator (TEO) was first described in \([14]\\). It can achieve amplitude and frequency estimation of an input signal using three samples without using QSG or causing ill-conditioning.

The Energy operator \( \Psi \) applied to a random signal \( x(t) \) is given by the following equation\([15]\\):

\[ \psi[x(t)] = \dot{x}(t) - x(t)\ddot{x}(t) \] \hspace{1cm} (14) \\

Applied to grid voltage \( v(t) \) described by (1) gives:

\[ \psi[v(t)] = V_m^2\omega^2 \] \hspace{1cm} (15)
D. Adjusting noise immunity

It can be noticed that the duration between two samples is not necessarily equal to one sampling period but it can also be a multiple of the sampling period to give to the method a better immunity against signal noises. For instance, giving \( N \) as the distance between two samples (such that the duration between two samples is \( NT \)), equation (9) can be generalized to the following formula:

\[
\hat{\omega}_{\text{CS}} = \frac{1}{NT} \cos^{-1}\left(\frac{v(n) + v(n-2N)}{2v(n-N)}\right)
\]

Increasing \( N \) provides a better noises immunity to the method, but slows its dynamic response down. Selecting \( N \) is a compromise between speed and robustness of the used method.

E. Comparison of the frequency estimation techniques

Modern frequency estimation techniques mentioned above can be separated into two groups: those who do not require a QSG (3CS, 4CS and TEO) and those who are based on QSG (ESTD, 2CS, E3CS and E4CS).

To compare the performances of those methods, a series of three tests is applied in simulation on each group. For all the three tests, the grid voltage is a unitary sine wave signal which shows a frequency jump from 50 Hz to 52 Hz at instant \( t = 0.3 \) s. For test 1, the input signal does not present any distortion. For test 2, the input signal is corrupted by a very light white noise, such that the signal-to-noise ratio reach 57 dB. For test 3, a third order harmonic of magnitude equal to 0.5% of the fundamental is added to the input signal. The sampling frequency is equal to 10 kHz.

The QSG used in this simulation is a frequency non-adaptive, transfer delay-based QSG (TD-QSG). This QSG relies on time-delaying the input signal using a buffer to create a quadrature component [4]. This way, neither the input signal nor the quadrature signal is filtered. Tests are performed with two distances \( N \) between samples. First, \( N \) equal 10, then \( N \) equal 30.

Results of test 2 show that increasing the sample distance enhances significantly the robustness of the frequency estimation techniques against noises. For the QSG-less techniques, the TEO method exhibits the weakest robustness against noises (Fig. 10). For the QSG-based techniques, results show

Fig. 7 shows simulation results of 3CS, 4CS, and TEO methods for test 1. It can be seen that the TEO method stands out of the other methods by the absence of notches due to ill-conditioning.

Fig. 8 shows simulation results of ESTD, 2CS, E3CS and E4CS methods for test 1. It appears that ESTD and 2CS methods present an oscillatory behavior after the frequency jump. This is due to the non-orthogonality between \( v_{\alpha} \) and \( v_{\beta} \) because the used TD-QSG is not frequency adaptive. Results of E3CS and E4CS methods are the same in steady-state, but E4CS method shows greater disturbances in transient operations. Those two methods are not sensitive to non-orthogonality between \( v_{\alpha} \) and \( v_{\beta} \).

Results of test 2 show that increasing the sample distance enhances significantly the robustness of the frequency estimation techniques against noises. For the QSG-less techniques, the TEO method exhibits the weakest robustness against noises (Fig. 10). For the QSG-based techniques, results show
that the E4CS is the most sensitive technique against noises, and the E3CS is the most robust one (Fig. 11).

![Figure 11: Estimated Frequency of QSG-based methods for test 2.](a) N=10, (b) N=30]

Results of test 3 (Fig. 12 and 13) shows that the TEO requires that the input signal has to be considerably filtered in order to reduce the estimated frequency distortions properly. Among all those techniques, the E3CS proves to be the most robust against low-order harmonics.

Tables 1 and 2 summarize features of the tested techniques. One can conclude that, according to its superior robustness, the E3CS method is the most suitable open-loop frequency estimation technique for grid synchronization.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Sensitivity to QSG unbalances</th>
<th>Robustness against noises</th>
<th>Robustness against low-order harmonics</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESTD</td>
<td>Yes</td>
<td>Moderate</td>
<td>Good</td>
</tr>
<tr>
<td>2CS</td>
<td>Yes</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>E3CS</td>
<td>No</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>E4CS</td>
<td>No</td>
<td>Moderate</td>
<td>Weak</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technique</th>
<th>Ill-conditioning issue</th>
<th>Robustness against noises</th>
<th>Robustness against low-order harmonics</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEO</td>
<td>No</td>
<td>Very weak</td>
<td>Moderate</td>
</tr>
<tr>
<td>3CS</td>
<td>Yes</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>4CS</td>
<td>Yes</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
</tbody>
</table>

![Figure 12: Estimated Frequency of QSG-less methods for test 3.](a) N=10, (b) N=30]

![Figure 13: Estimated Frequency of QSG-based methods for test 3.](a) N=10, (b) N=30]
IV. CONCLUSION

Open-loop synchronization techniques allow to overcome the Open-loop synchronization techniques overcome the stability issues inherent in closed-loop techniques. For single-phase power systems, all modern open-loop synchronization techniques, except the TEO, require a QSG to estimate the grid voltage amplitude and phase angle. However, this is not always the case for frequency estimation.

Most open-loop techniques differ in their QSG and frequency estimation methods. In this paper, an overview of the main frequency estimation methods was presented, along with a description of each technique and its structure for numerical implementation. A comparative study of these frequency estimation methods was conducted to test the robustness of each method. The results show that the E3CS method offers the best performance in terms of robustness against noise and low-order harmonics. Therefore, this method is recommended for single-phase power system applications.

REFERENCES


Hachemi Rahmani received his engineering degree in electronics from the University of Saad Dahlab in Blida in 2006, and then obtained his Magister degree from the Ecole Nationale Polytechnique in 2011. He joined the Solar Equipment Development Unit in 2012 as a Research Associate. His research area includes the control of grid-connected photovoltaic systems as well as data acquisition and graphical interfacing in distributed systems.

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