Analysis of Multiphase Electrical Systems According to the Number of Phases

Abdelhakim Khelafi, Abdelmalek Djebli, Omar Touhami, and Rachid Ibtiouen

Abstract−This paper studies multiphase electrical systems based on the properties of their number of phases. Depending on its parity, the latter can be even or odd. Its factorization provides numerous ways to represent the phase number. Thus, this paper presents the possibilities of modifications together with the properties that arise from the factorizations of the number of phases. Furthermore, we present the formulation of symmetrical components for both even and odd numbers of phases.

***Keywords*−Multiphase systems, multi-star systems, connections of Multiphase systems, symmetrical components, instantaneous symmetrical components.**

I. Introduction

The continuous increase in the demand,driven by significant industrial development, accounts for approximately 40% of global consumption [1]. Additionally, urban growth and the expansion of cities contribute to this trend by creating more complex and extensive infrastructure. This leads to a higher demand for electricity, which is essential for public sectors such as transportation and utilities, as well as for the growing needs of households. This makes an increase in electricity production inevitable. Therefore, upgrading and improving the efficiency of conventional methods of electricity generation and transmission has become an important requirement. Also searching for new sources of energy and better use of renewable energies. In addition, the consumption of electrical energy must be rationalized in the sectors that consume the most electricity which are mainly industrial and transport sectors, through the use of high efficiency systems. High-power electrical and electromagnetic systems are increasingly in demand in areas such as marine propulsion and electric traction. This demand is driven by the suitability of these systems, since electromagnetic systems are characterized by high reliability. Therefore, multiphase electrical systems MES have attracted significant interest from researchers due to their numerous advantages such as higher power density, higher fault tolerance [2].These advantages have led to an increased use of MES in the electric traction systems, marine propulsion, more electric aircraft, electrical vehicles and industry[3-8]. Multiphase electric power generators have obvious advantages over conventional three-phase generators, including the ability to achieve higher power at lower voltages [9], which reduces the level of winding insulation. In the literature, multiphase generators are not discussed for their use in conventional power plants (gas power plant, and combined cycle). However, they have been proposed as alternatives to their three-phase counterparts in renewable energy conversion systems [7]. Where the most important objective is to maintain a stable and continuous output power which will be transferred to the electrical networks. The number of phases (greater than 3) of multiphase generators offers to generator higherfaults tolerance and a higher degree of freedom, which allow the development of new controls in the faulty case. This makes multiphase generators suitable for wind power systems, especially variable speed ones, and suitable for marine power systems [10-13], [9]. Multiphase generators have also proven themselves in the field of aeronautics, electric vehicles and ships [14-15]. In aeronautics, the current trend is to replace hydraulic and mechanical systems in aircraft with other electrical and electromagnetic ones [16]. And it is an application of the concept known as the more electric aircraft [17]. The market for electric cars has also experienced enormous development, especially since the beginning of the second decade of the 21st century, due to the success recorded in the field of batteries. However, research is still ongoing to develop more reliable driving systems, and to increase the storage capacity of the batteries. The majority of studies on MES have aimed to deal with system performances, design, and control development. Most of them concluded that multiphase systems are appropriate for the applications requiring higher reliability, higher power density and fault tolerance. However, in the literature few works address the additional properties offered by an appropriate choice of number of phases. Hence, this work presents an analytical study of the advantages offered by the characteristics of the number of phases, which is a positive integer.

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A. Khelafi, A. Djebli, O. Touhami, and R. Ibtiouen are with Electical Department, Ecole Nationale Polytechnique, Algiers, ALGERIA. (e-mail: abdelhakim.khelafi@g.enp.edu.dz, abdelmalek.djebli@g.enp.edu.dz, omar.touhami@g.enp.edu.dz, rachid.ibtiouen@g.enp.edu.dz)

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II. Properties of the Number of Phases and the Modifiability of the Multiphase System

The concept of multiphase systems became important, after the advent of multiphase electric machines in the late 19th century [18], in which the electromagnetic quantities are rigidly linked to the multiphase windings of these machines. Since then, the three-phase systems are adopted for the production, transmission of electricity, transportation and industry applications. A symmetric multiphase system by definition is a set of *m* sinusoidal quantities of the same frequency and the same amplitude, where the phase difference between the adjacent phases is $2\pi/m$ [19]. The set of *m* quantities defined as MES, can be *m* currents, *m* voltages, or *m* magnetic fluxes. The name of a multiphase system is associated with its number of phases, in the English nomenclature, a multiphase system of m phases is called "m-phase", often m is in letter. In contrast, in French nomenclature, the name of a multiphase system with *m* phases consists of a Greek prefix followed by the word "phasé", the Greek prefix corresponds to Greek numbers. Examples of multiphase system nomenclatures are given in Table. I. There is however an exception in French nomenclature, when *m*=2, in this case the system is called "biphasé" [20].

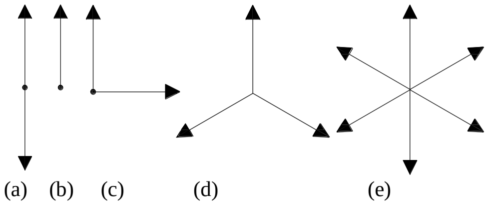

Fig.1 Representation of MES using phasor diagrams (a) two phase system (b) One-phase system (c) two-phase system the angle between the two phases is $\pi/2$ (d) three-phase system (e) six-phase system.

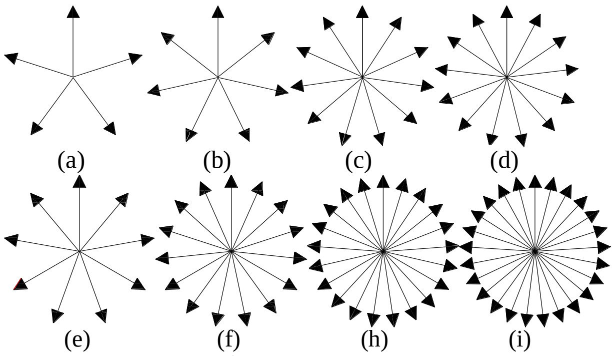

Fig.2 Phasor diagrams of multiphase systems with prime and odd phase numbers (a) Five-phase (prime) (b) Seven-phase (prime)(c) Eleven-phase (prime) (d) Thirteen-phase (prime) (e) Nine-phase (f) 15-phase (h) 21-phase (i) 25-phase.

The geometric representation of MES, known as phasor diagrams Fig.1, is the most suitable in the steady state [21]. Phasor diagram is a geometric tool allowing calculations such as addition, as well asdifferentiation and integration of sinusoidal functions of the same frequency. This makes them invaluable for analyzing MES, where they visually represent phase relationships and magnitudes of voltages or currents. Theoretically, the number of phases can be any integer number. Nevertheless, in the literature, there are few references dealing with the properties of multiphase electrical systems MES according to the number of phases *m*, including the most interesting [21-25]. Thus, based on [21-23] the MESs can be classified according to the nature of their number of phases *m*. This classification distinguishes MESs with even *m*, odd *m*, and prime number *m* of phases, each representing distinct configurations that contribute to understanding the varied characteristics and behaviors of MESs. If the phase number *m* of a multiphase system equals two ($m=2$), then *m* is both even and prime. By applying the previously mentioned definition, the phase angle between the two phases is π Fig.1 (a), this system is said to be two-phase [21]. It can be seen that the representation Fig.1 (a) of this system consists of two vectors, each vector represent one phase as shown in Fig.1 (b), these vectors having the same axe, whereas their directions are opposite.Thus, in the case where this system represents machine, its winding consists of two coils with an angle of π between their axes.The field lines are parallel, and hence, no rotating magnetic field is produced [21]. The system shown in Fig1(c) is also a two-phase system, nevertheless these two phases are in quadrature, the phases are shifted by π/2, in English nomenclature such a system is called Two-phase system. It should also be noted that the two-phase system is a particular case of the four-phase system [20]. Two-phase systems are still used, for example Scott's transformer in high-speed train. Currently, two-phase machines are not used, however the principle of two-phase machines is used in single-phase machines, where an auxiliary winding shifted by π/2 is introduced. This winding is usually connected to a capacitor called a start capacitor, which generates the voltage phase shift necessary to create rotating field, thereby producing the electromagnetic torque. The MESs with an even number of phases can be seen as being composed of m/2 two-phase systems (as shown in Fig.1 (a)) displaced by π/m, such as the six-phase system in Fig.1 (e) which is formed by three two-phase systems. In practice, this system can be realized with a three-phase transformer, where each secondary phase coil has midpoint. According to the above, MESs can be classified according to the nature of the number of phases into three categories: MES with an even number of phases, MES with an odd number of phases and MES with a prime number of phases.

A. Multiphase System with Odd Number of Phases

The theory of arithmetic states that every odd number m can be uniquely factored into a product of prime factors different from two. Moreover, a prime number cannot be factorized. Figure (2) shows phasor diagrams of MESs with number of phases odd, and prime, the systems in Fig.2 (a) (b) (c) (d) are with prime number of phases which are 5, 7, 11 and 13 respectively, the systems in Fig.2 (e) (f) (h) (h) are with non-prime odd number of phases which are 9, 15, 21 and 25 respectively .
Thus, *m* is written as (1).

$$m=s_1\times s_2\times\ldots\times s_k \qquad (1)$$

Where s_1, s_2, s_k are non-two prime numbers, and *k* is an integer number.
According to (1), there are several manners to express m and therefore, several ways of seeing the MES. The most appropriate m expression is the product of two numbers, because this allow to analyze the system properly. Thus, by rewriting (1) assuming $m_{ph} = s_1$, and $s = s_2 \times \ldots \times s_k$, the equation (2) is obtained.

$$m= s \times m_{ph} \qquad (2)$$

According to the expression (2) the MES of *m* phases can be considered formed by s multiphase systems of m_{ph} phases, where the angle shift is 2π/ m. it is possible to assume $s=s_1$ and$m_{ph}= s2\times\ldots\times s_k$. For example, a 15-phase MES, where $m=15$ is the product of three and five (m=3×5). Thus, the 15-phase MES illustrated in Fig.2 (f)can be seen as formed either by three five-phase systems shifted by 2π/15 or by five three-phase systems shifted by 2π/15. Also the nine-phase MES in Fig.2 (e) can be seen formed by three three-phase systems shifted by 2π/9.Hence, a MES with non-prime odd number of phases *m*, as expressed by (2) is modifiable. This modification depends on how the MES is viewed. If the MES is viewedas being formed by *s*MESs of m_{ph} phases. Thus, by rotating the (s-

Table. I
NOMENCLATURE OF MULTIPHASESYSTEMS.

Phase number m	Greek Prefix	French nomenclature	English nomenclature
1	mono	monophasé	one-phase
2	-	-	-
3	tri	triphasé	three-phase
4	tétra	tétraphasé	four-phase
5	penta	pentaphasé	five-phase
6	hexa	hexaphasé	six-phase
7	hepta	heptaphasé	seven-phase
8	octa	octaphasé	eight-phase
9	nona	nonaphasé	9-phase
10	déca	décaphasé	10-phase
11	undéca	undécaphasé	11-phase

1) systems of m_{ph} phases by π, this results in a multi-star system formed by s m_{ph}-phasesystems. Otherwise, if the MES is seen formed by $m_{ph}$$s$-phase systems, by rotating the (m_{ph}-1) the s-phase systems by π, this results in a multi-star system formed by $m_{ph}$$s$-phase systems. However, the total number of phases is still$m$, the number of phases of the original system.

It has been previously mentioned that a 15-phase system canbe seen as formed either by three five-phase MESs shifted by 2π/15 or by five three-phase MESs shifted by 2π/15, as depicted in Fig. 3 (a) (b), respectively. Therefore, by rotating two five-phase MESs in Fig.3 (a) by π, which corresponds to the inversion of the corresponding winding terminals, a triple-star five-phase MES is obtained as shown in Fig.3 (c). The 15-phase MESFig. 3 (b) can be transformed into MES of five star three-phase system as shown in Fig.3 (d), this by rotating four three-phase systems of Fig.3 (b) by π.

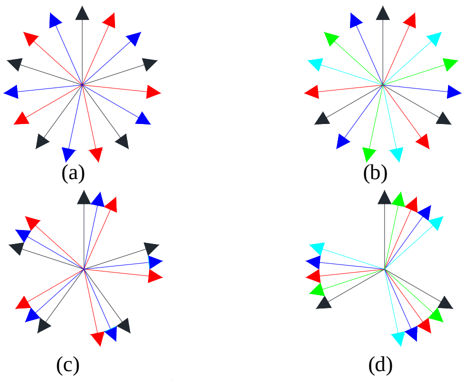

Fig.3 Modification possibility of MES with odd non-prime number of phases (a) 15-phase system formed by three five-phase systems (b) 15-phase system formed by five three-phase systems (c) Triple -star five-phase system (d) Five-star three-phase system.

B. *Multiphase System with Even Number of Phases*

In this case, the number of MES phases m can be expressed by (3), four-phase, six-phase eight-phase and twelve-phase MESs are shown in Fig. 4 (a) (b) (c) (d), respectively. Where m_{ph} in (3) is an integer. The even number m of phases can be halved [21], [22] and becomes m_{ph} phases. Such a system can be considered formed by two m_{ph}-phase MSEs shifted by π. Depending on the nature of m_{ph}, there are two cases: the first is when m_{ph} is odd, and the second is when m_{ph} is even. The usefulness of this writing will be seen later. Practically, reduction of the number phases is performed by reconnecting the terminals of the windings [22], [23]. In the case of transformers and machines, for example a three-phase transformer where the secondary is with midpoints, this secondary is a six-phase system shown in Fig.4 (b). Therefore, applying the concept of the number of phase reduction, the six-phase system of transformer secondary Fig.4 (b) can be reduced to three-phase Fig.1 (d) by reversing the terminals and connecting them in series.

$$m=2.m_{ph} \qquad (3)$$

C. *Reduced Multiphase System*

The concept of phase number reduction came from multiphase electrical machines [21-23], it is the winding of the machine that enables the reduction of the number of phases, without rewinding.This is achieved through the modification of the connections of coil groups [21]. In practice, the windings of conventional three-phase electrical machines are originally six-phase windings [21-22] that have been reduced to three-phase.This reduction in the number of phases achieved by appropriately reconnecting the groups of coils. Additionally, this modification increases the winding factor from 0.85 to 0.96. Furthermore, reducing the number of phases from 6 to 3, maintains the same spatial harmonics content as the original 6-phase winding. It should be noted that this phase number modification is only possible for certain windings with an even initial phase number m and a phase shift of 2π/m [24].

Since the even number of phases m is expressed by (3), two cases can be distinguished based on the nature of the number m_{ph}: the first where m_{ph} is even and the second m_{ph} is odd. Moreover, when m_{ph} is an odd prime number, this represents a

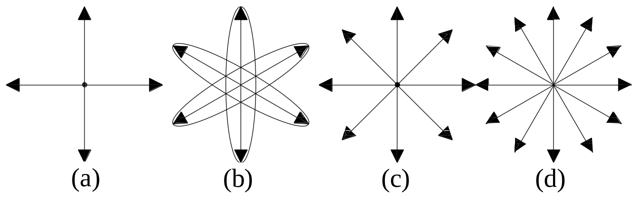

Fig.4 Phasor diagrams of multiphase systems with even number of phases (a) four-phase (b) six-phase (c) eight-phase (d) twelve-phase.

special case.

D. *Reduced Multiphase System with m_{ph} Odd Number*

In this case, the m-phase MES is formed by two m_{ph}-phase MESs shifted by π. Geometrically, the phase number reduction consists of rotating one of the two m_{ph}-phase systems by π. In practice, the reduction consists of inverting one of the two m_{ph}-phase windings, and putting them either in series or in parallel. Thus, the phase number m of MES is halved and becomes m_{ph}, and the angle shift between two adjacent phases becomes $2\pi/m_{ph}$.

Therefore, an electrical machine winding with a reduced number of phases at m_{ph} has the identical harmonic content and winding factor as the winding of 2×m_{ph} phases. For example a three-phase winding is actually 6-phase reduced, a five-phase is a reduced 10-phase winding and a seven-phase winding is a 14-phase winding reduced.

E. *Reduced Multiphase System with m_{ph}Even Number*

In this case, the number of phases m is even represented by (3), in addition m_{ph} in (3) is also even expressed by (4). Therefore, all the vectors representing the phases are located in the same semicircle [24]. Moreover, if the number m'_{ph} in (4) is odd, the adjacent phases are shifted by $(\pi/2)/m'_{ph}$. The reduction is achieved by inverting the vectors in the third and fourth quadrants of the original MES phasor diagram [24]. Consequently, the reduced MES is formed by m'_{ph} two-phase systems [24]. The Figure 5 (a) shows the reduced 12-phase SEP (12=2× 2×3), m'_{ph} =3, the reduced MES is formed by three two-phase MES shifted by π/6 Fig.5 (a).
In the general case when the number m_{ph} is an even multiple of an odd number, there exists an odd number m'_{ph} such that m_{ph} can be expressed as (5), where k in equation (5) is an integer with $k \geq 1$. Consequently, the reduced MES consists of $2^{k-1} \times m'_{ph}$ two-phase systems, shifted by $\pi/(2^k m'_{ph})$.

In a particular case where the number of phases m is expressed by (6) with $k \geqslant 2$, the reduced MES is formed by $(k-2)$ two-phase systems, phase shifted by $\pi/2^{k-1}$.For example, reducing a 4-phase system yields a two-phase system as shown in Fig.5(b), reducing an 8-phase system yields two two-phase systems shown in Fig.5(c), and reducing a 16-phase system yields four two-phase systems illustrated in Fig.5(d).

$$m_{ph}=2.m'_{ph} \quad \text{where } m'_{ph} \text{ is odd number} \qquad (4)$$

$$m_{ph}=2^k.m'_{ph} \qquad (5)$$

$$m=2^k \qquad (6)$$

Where$m'_{ph}$is an odd number

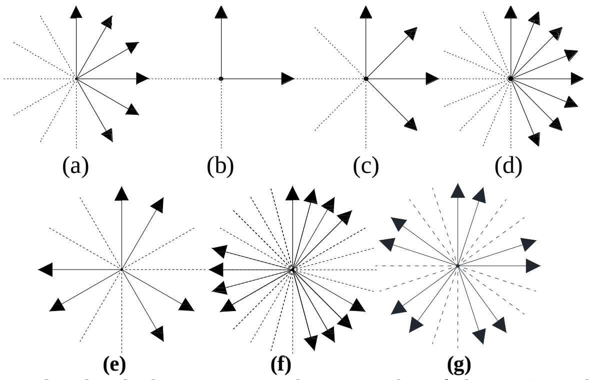

Fig.5 Reduced multiphase systems with even number of phases (a) 12-phase reduced to three two-phase(b) 4-phase reduced to 2-phase system (c) 8-phase reduced to two 2-phase (d) 16 –phase reduced to four two-phase (e) 12-phase reduced to double star three-phase (f) 24-phase reduced tofour three-phase system (g) 20-phase system reduced to double star five-phase system.

F. Multi-Star Multiphase System

If m_{ph} is expressed by (4), where m'_{ph} is odd, an additional phase reduction is possible beyond the previously mentioned phase number reduction. In this case, the MES is considered to consist of twom_{ph}-phase systems shifted by π/m_{ph}.Hence, the reduction of each m_{ph}-phase system results in a reduced system consisting of two m'_{ph}-phase systems shifted by $\pi/(2\, m'_{ph})$.This configuration is known as a double star system. Therefore, m-phase MES with m is an even number of phases, as presented in (3), when the number m_{ph} in (3) is expressed by (4).In this case, the *m*-phase systemcan be reduced and transformed into a double star system consisted of two m'_{ph}-phase systems shifted by $\pi/(2\, m'_{ph})$.In general case whenm_{ph} is written as (5), the m-phase system can be reduced and transformed into a multi-star system formed by $2^k m'_{ph}$-phase systems shifted by $\pi/(2^k m'_{ph})$.

The Figure 4 (d) shows a 12-phase MES (12=2×2×3), $m'_{ph}=3$, hence, it is possible to obtain a double star MES formed by two stars of three-phase shifted by π/6 Fig.5 (e). This system called asymmetric three-phase system, and its application in asynchronous machines is widely discussed in the literature. Therefore, the number of phases in such machine is actually a 12 phases, the corresponding winding is reduced by reconnecting the coil groups. This winding is generally used in motors supplied by two three-phase inverters their three-phase voltages are shifted by π/6. The reduction in the number of phases from 12 to two shifted three-phase, allows the use of conventional three-phase inverters. In addition, this winding has the same performance (high winding factor and reduced harmonic content) as a 12-phase winding. A 24-phase system can be reduced and transformed into four star three-phase system as shown in Fig.5 (f). Similarly,a 20-phase system can be reduced and transformed into double star system formed by two five-phase systems Fig.5 (g).

G. Reduction of Multi-Star Systems

A multi-star system, where the star represents a symmetric multiphase system of odd number of phases m'_{ph}. In multi-star systems, the vectors in the phasor diagram are divided into groups. Thus, by representing each group of vectors by their resultant vector, a MES of m'_{ph} phases is obtained. Consequently, the number of phases in the multi-star system can be reduced to match the phase number of a single star.

Furthermore, m-phase system*m*with number is an odd non-prime, its number of phases can be reduced. This by transforming the m-phase system into multi-star system, and represent each group of victors by their resultant as mentioned previously. The case of nine-phase transformed into three-phase triple star and reduced to three-phase is presented in Fig.6.Similarly, a 15-phase system can be transformed into a five-phase three-star system and reduced to five-phase system, as illustrated in Fig. 6.

H. Particularity of Multiphase System with a Prime Number of Phases

Multiphase systems with number of phases *m*that prime numbers are not modifiable and cannot be reduced. For example, a six-phase transformer secondary shown by the phasor diagram Fig.4 (b) and a three-phase double-star machine represented by the phasor diagram Fig. 5 (e), these systems can be reduced by reconnecting the winding terminals [22] and become three-phase systems. However, the three-phase secondary of a transformer or a three-phase machine cannot be modified or reduced [21].

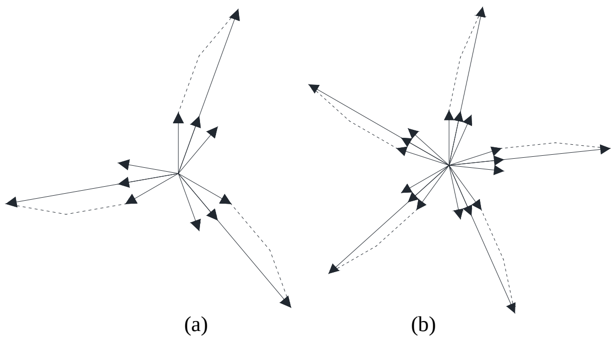

Fig.6 Reduction of multi-star systems (a) Reduction triple- star three-phase to three-phase (b) Reduction triple- star five-phase to

Therefore, an m-phase system (with m prime number) represents a basis for n-phase systems (n multiple of m) and for multi-star of m-phase systems. Thus, the study of m-phase system facilitates the study of n-phase systems (n multiple of m) and for multi-star m-phase systems, where the number of phases is high.

In addition, multiphase systems (m prime number) are advantageous in terms of harmonic content.

III. Multiphase System Connections

Phase connections are primarily for multiphase systems with multiphase windings, multiphase such as machines and multiphase transformers. The windings of the phases in a *m*-phase machine or transformer have 2×m terminals. Consequently, several connections of the windings of the *m* phases are possible. Each connection is characterized by the voltage resulting across a phase winding terminals. Depending on the parity of the phase number, two cases are distinguished: the first when*m* is odd and the second when*m* is even. Nevertheless, the star and polygon couplings independent of the parity of the number of phases are obvious connections.

In a star connection, thecharacteristic voltage is the phase voltage V_{ph} of the multiphase system. In contrast, a polygon connection is characterized by the voltage between two adjacent phases. The equation (7) expresses the voltage between adjacent phases U_{ad} as a function of the number of phases*m* and the phase voltage V_{ph}.

If the number of phases *m* is odd, the number of possible connections is *(m+1)/2*. By subtracting the two obvious connections star and polygon, the result is (m-3)/2 connections between non-adjacent phase windings.

The five-phase and seven-phase systems, which have prime numbers of phases, are illustrated in Fig. 7 and Fig. 8, respectively. Each connection, except for the star connections, is depicted as a single regular polygon, as shown in Fig. 7 (b) (c), Fig. 8 (b) (c) (d). The connection in Fig.7 (c) is called pentacle connection which is a regular pentagram characterized by the voltage between non-adjacent phases. The connections in Fig. 8 (c) and (d) are both regular heptagrams. Therefore, in the case of an *m*-phase winding where *m* is a prime number, the possible connections are star and polygon connections, in addition to ((m+1)/2)-2 regular polygram connections.

The connections of a nine-phase system, shown in Fig. 9, include star Fig. 9 (a), nonagon Fig. 9 (b), and two regular Fig. 9 (c) Fig. 9 (d). In this case, the number of phases is a non-prime odd number and can be expressed as (2). Hence, a particular connection is formed by three triangles shifted by 2π/9, as shown in Fig. 9 (e).

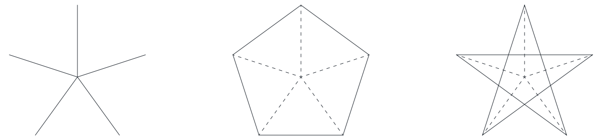

Fig.7 Connections of five-phase system (a) star connection (b) Pentagon connection (c) Pentacle connection

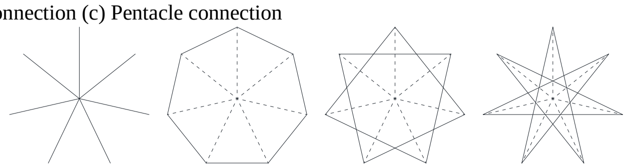

Fig.8 Connections of seven-phase system (a) star connection (b) Heptagon connection (c) and (d) Tow regular Pentagram connections

In m-phase windings with an odd non-prime number of phases, as expressed by equation (2) where m_{ph} and *s* are prime numbers, the connections include two particular connections. Each connections consists of distinct polygons: the first consisting of *s*polygonswith m_{ph} sides,shifted from each other by 2π/m, and the second consisting of m_{ph} polygons with *s* sides, shifted from each other by 2π/m. The connections are more complicated when m_{ph} and/or *s* are odd non-prime numbers.

Regarding the connections of systems with an even number of phases, the systems in question are those with four or more phases. This is because systems with two phases have only one possible connection. In four-phase systems, there are two connections: the star connection and the square connection. In systems m-phase with *m*>4, the connections include: star and polygon configuration, polygram connections, and arrangements consisting of multiple shifted polygons.

Table. II

RATIO OF PHASE-TO-PHASE VOLTAGES OF ADJACENT AND NON-ADJACENT PHASES TO PHASE VOLTAGE

Ratio U_k/V_{ph}	Nombre de phases m							
	m=5	m=6	m=7	m=8	m=9	m=10	m=11	m=12
Adjacent Phases	1.17	1	0.87	0.76	0.68	0.61	0.56	0.52
Non adjacent Phases	1.9	1.73	1.56	1.41	1.28	1.17	1.08	1
	-		1.95	1.85	1.73	1.61	1.51	1.41
	-	-	-	-	1.97	1.9	1.82	1.73
	-	-	-	-	-	-	1.98	1.93

When the number of phases *m* is even, the number of possible connections is m/2. Therefore, the number of non-adjacent voltages is (m/2-2), the possible connections of 8-phase and 10-phase systems are shown in Fig.10 and Fig.11 respectively.

Since each connection is characterized by a voltage, the number of possible voltages corresponds to the total number of connections. These voltages include phase voltage, voltage between adjacent phases, and the voltages between non-adjacent phases. U_{ad}

The generalized formula giving the voltages between adjacent and non-adjacent phasesU_k is expressed by (8), the voltages between non-adjacent phases are obtained from (8) when *k>1*.

The Table. II gives the U_k/V_{ph} ratio in different cases where

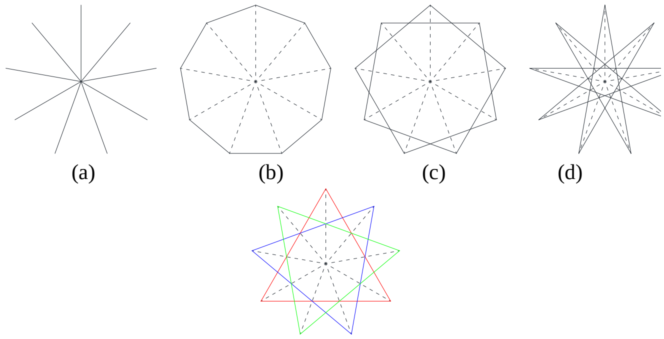

Fig.9 Connections of nine-phase (a) star connection (b) Nonagon connection (c) and (d) Tow regular nonagram connections (e) connection formed by three separated trianglesshifted by 2π/9.

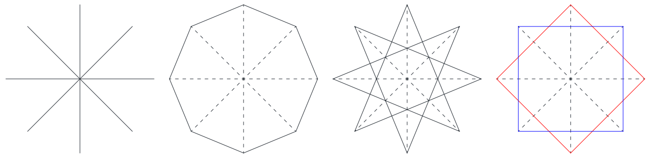

Fig.10 Connections of eight-phase system(a) star connection (b) Octagon connection (c) regular octagram connection(d) Connection is formed two square shifted by π/2.

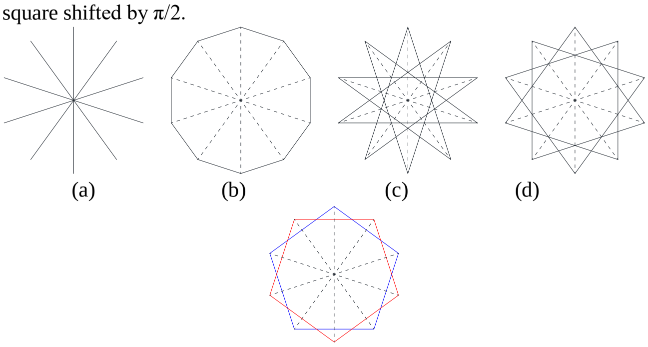

Fig.11 Connections of 10-phase system (a) star connection (b) decagon connection (c) (d) Two regular decagram connections (e)connection formed by two separated pentagons shifted by 2π/10.

$1 \leq k \leq m$,is evident that as the number of phases increases, the voltage U_{ad} between adjacent phases given by (7) decreases, resulting in a reduced ratio U_{ad}/V_{ph}. For instance, in a five-phase system, the ratio is 1.17, whereas it drops to 0.87 in a seven-phase system and further decreases to 0.52 in a twelve-phase system. In contrast, the voltages between non-adjacent phases U_k (*k>1*) increase with the number of phases.For example, in the five-phase system, the ratio is 1.9, compared to 1.93 in the 12-phase system.

$$U_{ad} = 2.V_{ph}.\sin\left(\frac{\pi}{m}\right) \quad (7)$$

$$U_k = 2.V_{ph}.\sin\left(\frac{k\pi}{m}\right), \begin{cases} m: odd..k = 1,...,(m-1)/2 \\ m: even..k = 1,...,m/2-1 \end{cases}, \quad (8)$$

IV. METHOD OF SYMMETRICAL COMPONENTS

The method of Symmetrical Components SC is considered the most suitable method for the analysis of multiphase systems, was presented for the first time by C. L. FORTESCUE 28 Jun 1918 at the 34th annual convention of the American Institute of the Electrical Engineers in Atlantic City (NJ, USA), where he introduced his paper [19]. The method of symmetric components had its origin in the mathematical analysis of induction machines operating under unbalanced conditions

[22]. The purpose of this method as indicated by C. L. FORTESCUE in [19] is to study multiphase systems (multiphase machine, multiphase network, etc.) under unbalanced conditions. In his article [19], C. L. FORTESCUE refers to this method by several names: symmetrical co-ordinates, sequence components. Currently, this method is known as "the method of Symmetric Components".

In 1913, Fortescue conducted a mathematical study of the behavior of asynchronous motors under unbalanced conditions. What caught the author's attention the most in the results obtained was their symmetry [19]. The solution always

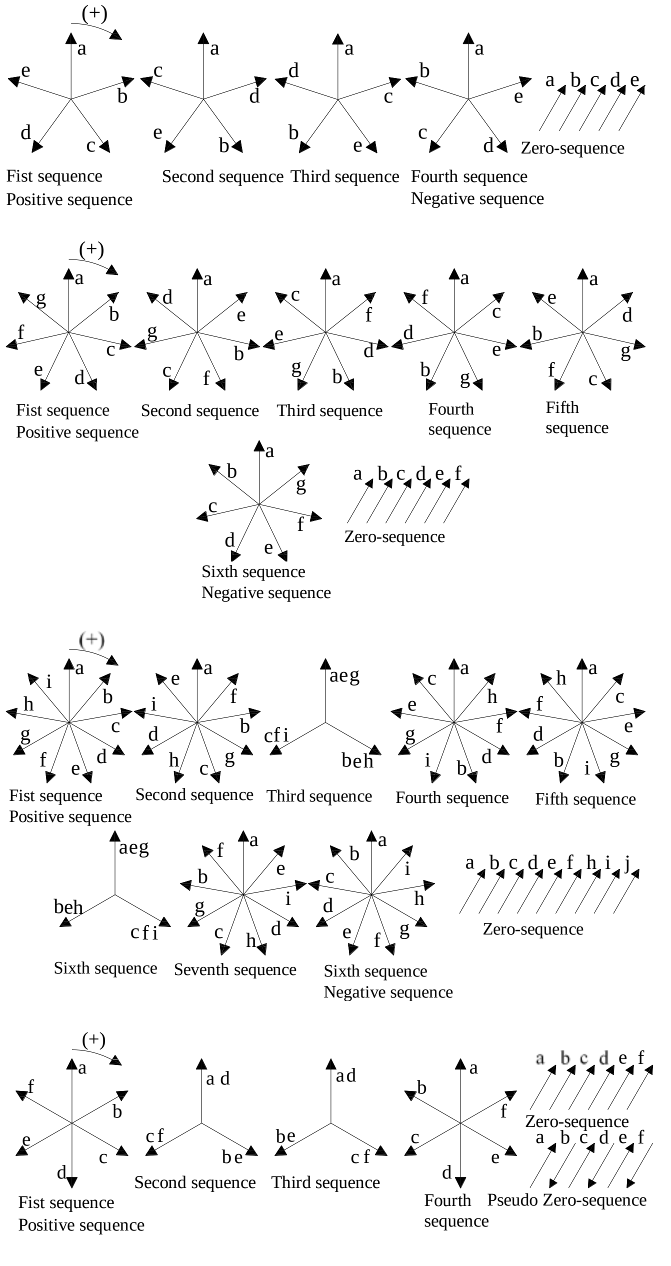

Fig.12 Symmetrical components of five-phase, six-phase, seven-phase and nine-phase systems.

reduced to the sum of two or more symmetric solutions [19]. This then leads the author to ask whether there are general principles by which the solution of unbalanced multiphase systems can be reduced to the solution of two or more balanced multiphase systems [26-27].

The great advances in the application of fundamental laws to electrical circuits are due to the development of mathematical tools. It is the same for the SC, since the original formulation of the SC presented in [19] was written in the form of an equation system containing many equations representing the phases of a MES. This method has been rooted mathematically, reformulating it in matrix form. What makes SC an object of algebra [27], where algebraic concepts such as the concept of passage from one space to another using passage matrices; the passage matrix is known in the literature by transformation matrix. Additionally, the algebraic formulation made SC easy to understand and use. This has made it the main method for calculating MESs, in particular three-phase systems which are the most used in machinery and transmission and distribution networks. Furthermore, the algebraic formulation allowed to derive the ubiquitous Clarke and Park transformations from the Fortescue symmetric component transformation [27].

Since the publication of [19] at the beginning of the 20th century, numerous research articles have explored the applications of SC in the study and modeling of both synchronous and asynchronous three-phase machines, as well as three-phase networks [27-30]. Furthermore, multiphase systems with a number of phases m are discussed [21-24], in which two cases are distinguished: the first when$m$ is odd and the second when$m$ is even.

The transformation of the m quantities of m-phases MES,where m is an odd number, into symmetrical components SC is expressed by (9). The transformation of the quantities of an m- phase, where m is an odd number, into symmetric components SC, is expressed by (10). The transformation matrices in the case where$m$ odd and m is even are represented byC_s and C_{s2}, as shown in equations (11) and (12), respectively. The quantities X_k ($k=1.2\ldots m$) in (9) and (10) are the phase quantities which can be currents, voltages…

$$\begin{bmatrix} X_{cs0} \\ X_{cs1} \\ X_{cs2} \\ X_{cs3} \\ \vdots \\ X_{cs(m-1)} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & a & a^2 & a^3 & \cdots & a^{(m-1)} \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ \vdots \\ X_m \end{bmatrix} \quad (9)$$

etc.

Where $a=e^{j2\pi/m}$, and the phase quantities are expressed by vector X, while the symmetrical components are expressed by the vector X_{cs}.

When$m$is odd, the quantities X_{cs0} and X_{csk} ($k=1, 2\ldots (m-1)$) are the symmetrical components of the m-phase system. The componentX_{cs0} is called the zero sequence component. The components X_{cs1} and $X_{cs(m-1)}$ are the positive sequence components and the negative sequence component respectively.

In the case of an even number of phases, the symmetric components of the system are X_{cs00}, X_{cs0} and X_{csk} ($k=1, 2\ldots (m-2)$). The components X_{cs0}, X_{cs1} and $X_{cs(m-1)}$ are the zero sequence, positive and negative sequences. The component$X_{cs00}$is referred to as the pseudo-zero component. This pseudo-zero component does not exist in systems with an odd number of phases [29]. On the other hand, systems with an even number of phases contain the pseudo-zero component [27].

Each symmetrical component, with the exception of the zero-sequence and pseudo-zero sequence components, represents a symmetrical multiphase system of different phase succession [19], [30]. In the case of five-phase system, the succession of phases**abcde** corresponding to the positive sequence component while the succession of phases**edcba**is corresponding to the negative sequence component. The phase succession **acebd** is corresponding to the second sequence component, and the succession**adbec** is corresponding to third sequence component, this succession of phases is the inverse of that corresponding to the second sequence component.

The symmetrical components of five-phase system, where the number of phases is prime,represent four symmetrical five-phase systems and one zero-sequence as shown in Fig. 12 (a). Likewise, thesymmetric components of a seven-phase system, with prime number of phases,represent six symmetrical seven-phase systems and one zero-sequence component, as shown in Fig. 12 (b).In the case of the nine-phase system, where the number of phases is an odd non-prime number, the symmetrical components represent one zero-sequence component, five nine-phase symmetrical systems, and two particular components each representing three-phase system, as shown in Fig.12 (c).The figure 12 (d)illustrates the representation of the six components of a six-phase system, including two six-phase systems, two three-phase systems, one zero-sequence component and one pseudo-zero-sequence component.

Hence, it is possible to generalize that in the case of an m-phase system with a prime number of phases, the m symmetrical components represent (m-1) m-phase symmetrical systems and one zero-sequence system. In the case of an odd non-prime number of phases, the symmetrical components represent one zero-sequence component, at least two m-phase symmetrical systems, and several symmetrical systems with a phase number smaller than m. In the case of an even number of phases, the symmetrical components represent two m-phase systems, zero-sequence and pseudo-zero components, and (m-4) systems with a smaller number of phases than m.

The matrices C_s and C_{s2}defined in equations (11) and (12) respectively,are the transformation matrices. The matrixC_s is transformation matrix when m is odd, and C_{s2} is the transformation matrix when m case is even.Both matrices C_s and C_{s2} are $m\times m$ square matrices. Algebraically, these matrices are called change-of- basis matrices, these matrices allow to express the quantities of phases by symmetrical components.

$$\begin{bmatrix} X_{cs0} \\ X_{cs00} \\ X_{cs1} \\ X_{cs2} \\ \vdots \\ X_{cs(m-2)} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & 1 & -1 & \cdots & -1 \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ \vdots \\ X_m \end{bmatrix} \quad (10)$$

$$C_s = \frac{1}{m} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & a & a^2 & a^3 & \cdots & a^{(m-1)} \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix} \quad (11)$$

$$C_{s2} = \frac{1}{m} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & 1 & -1 & \cdots & -1 \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix} \quad (12)$$

The computation of the inverse matrices of C_s or C_{s2} is simple by applying (14) [29]. The matrix $C_s^{-1}$the inverse of C_s is given by (15).

$$C_s^{-1}=m\times C_s^{*T} \quad (14)$$

Furthermore, the transformation matrices can be rewritten to obtain unitary transformation matrices, the case of the matrix C_{su} expressed by (16). The unitary form offers the advantage that the power and torque in a machine, or power in a transformer, resulting from the transformed voltages and currents using a unitary matrix, are invariant [29-30].

$$C_s^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & a^{-1} & a^{-2} & a^{-3} & \cdots & a^{-(m-1)} \\ 1 & a^{-2} & a^{-4} & a^{-6} & \cdots & a^{-2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & a^{-(m-1)} & a^{-2(m-1)} & a^{-3(m-1)} & \cdots & a^{-(m-1)(m-1)} \end{bmatrix} \quad (15)$$

$$C_{su} = \frac{1}{\sqrt{m}} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & a & a^2 & a^3 & \cdots & a^{(m-1)} \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix} \quad (16)$$

A. Symmetric Components in Steady state

The subject of SC study focuses on the MES in steady state, where the MES quantities often assumed to be sinusoidal, and are represented by their amplitudes and phase shifts. In general, MESs are represented as complex numbers, regardless of whether they are balanced or unbalanced, as expressed by (17). This calculation method is still employed for determining short-circuit currents in networks and estimating imbalances in MESs, particularly in networks [26-27]. Its continued use is due to its simplicity and its capacity to facilitate the understanding of MES operation in unbalanced conditions.

Moreover, this writing (17) of sinusoidal quantity is easy to represent by the phasor diagrams previously mentioned.

In the equation (17) X_k and ϕ_k are the amplitude and the phase

$$X = \begin{bmatrix} X_1.e^{j\phi_1} \\ X_2.e^{j\phi_2} \\ X_3.e^{j\phi_3} \\ \vdots \\ X_m.e^{j\phi_m} \end{bmatrix} \quad (17)$$

shift respectively, the index $k = 1, 2 \ldots m$.

In a symmetrical (balanced) MES the amplitudes are equal, and the phase shift angles are written in this form: $k2\pi/m$, where $k = 0,1, 2, \ldots (m-1)$. So, the quantities of a symmetric MES are expressed by (18).

$$X = \begin{bmatrix} X \\ X.e^{j\frac{2\pi}{m}} \\ X.e^{j\frac{4\pi}{m}} \\ \vdots \\ X.e^{j\frac{(m-1)2\pi}{m}} \end{bmatrix} \quad (18)$$

The symmetric components, in steady state, of a multiphase system of phase quantities represented by (17), are calculated using one of the two transformations (9) or (10), depending on the parity of the number of phases. Therefore, the SCs, in the case odd m $X_{cs1},\ldots, X_{cs(m-1)}$, represent (m-1) symmetric multiphase systems, and the one zero sequence component which is vector characterized by its calculated amplitude and phase shift, generally the zero sequence component is represented by m vectors in parallel. In the case even m, the SCs $X_{cs1}\ldots, X_{cs(m-2)}$, represent (m-2) symmetric multiphase systems, one zero-sequence component and one pseudo-zero sequence component. The X_{cs1} component is called the positive component or positive sequence (in both cases m odd and even), this represents a symmetrical multiphase system where the sequence of vectors is counterclockwise. In the odd m case, the component $X_{cs(m-1)}$ is called the inverse component or inverse sequence. It represents a symmetrical multiphase

$$X=\begin{bmatrix} \sum_{\nu=0}^{+\infty} X_{(2\upsilon+1)} \\ \sum_{\nu=0}^{+\infty} X_{(2\upsilon+1)} e^{(2\nu+1)\frac{2\pi}{m}} \\ \sum_{\nu=0}^{+\infty} X_{(2\upsilon+1)} e^{(2\nu+1)\frac{4\pi}{m}} \\ \vdots \\ \sum_{\nu=0}^{+\infty} X_{(2\upsilon+1)} e^{(2\nu+1)\frac{(m-1)2\pi}{m}} \end{bmatrix} \quad (21)$$

Table.III

Harmonic rank	Sequence
vm	Zero sequence
2vm+1	Direct Sequence (contains the Fundamental sequence)
2vm-1	Sequence *(m-1)* called Inverse Sequence
(2v+1) m+2	Sequence 2
(2v+1) m+4	Sequence 3
:	:
(2v+1) m+2(k-1)	Sequence k
:	:
(2v+1) m+2[(m-1)/2-1]	Sequence *(m-1)/2*
(2v+1) m-2[(m-1)/2-1]	Sequence (m+1)/2, is the inverse of the Sequence (m-1)/2
:	:
(2v+1) m-2(k-1)	Sequence (m-k) is the inverse of Sequence k
:	:
(2v+1) m-2	Sequence (m-2) is the inverse of Sequence 2
1≤k≤ (m-1), and *v≥ 0*	

system where the sequence of vectors is clockwise. In the even case m the inverse sequence is $X_{cs(m-2)}$. The sequence of vectors of a symmetric multiphase system which represents a symmetric component, is said to be direct or inverse according to the clockwise direction.This depends on the transformation matrix C_s, particularly depends on the operator a, if $a=e^{j2/m\pi}$, the forward sequence is characterized by clockwise, and the reverse sequence is characterized by counterclockwise. In contarst, if $a=e^{-j\pi 2/m}$, the forward sequence is characterized by counterclockwise, and the forward sequence is characterized by clockwise.

B. Applications of Symmetrical Components to the Analysis of Harmonics

Usually, the waveforms of the quantities of MESs such voltages, currents, and fluxes, are not purely sinusoidal, leading to the appearance of harmonics. The quantities of the MESs are alternating quantities [26-27], and are generally considered symmetrical with respect to the time axis, which translates mathematically to being represented by odd functions. Consequently, the Fourier series expansion of these functions contains only terms of odd rank.

A function f: ℝ→ℝ, is said to be odd function if f satisfies condition (18).

$$f(-x)=-f(x),\ x\in\mathbb{R} \quad (18)$$

The development of the odd function $f$in a Fourier series is presented in equation (19).

$$f(x)=\sum_{\nu=1}^{+\infty} E_{(2\upsilon+1)}\cos\left((2\nu+1)x+\phi_{\upsilon}\right),\nu\in\mathrm{N} \quad (19)$$

Hence, the most common harmonics (of currents and voltages) in three-phase systems are odd-order harmonics. This can be generalized in the case of MESs, where representation (20) of the phase quantities of MESs is more general, because in practice even a filtered signal it remains contains a certain amount of harmonics.

Using equation (19), the phase quantity of an MES can be expressed as shown in equation (20).

$$X_k=\sum_{\nu=1}^{+\infty} X_{(2\upsilon+1)} e^{(2\nu+1)\frac{k2\pi}{m}},0\le k\le(m-1),(k,\nu)\in\mathrm{N} \quad (20)$$

As a result, thequantities of the phases of m-phase system are given by (21), m assumed to be odd.

For an odd number of phases, the SCs of the MES represented by (21) are determined using the transformation (9). Thus, the transformation of the MES (21) into symmetric components is expressed by (22).

The transformation (22) of MES (21) into SC results in symmetrical components having specified harmonic orders. Thus, the transformation (22) creates a classification of the existing harmonics in each symmetrical component. The classes of the harmonics of the symmetrical components are listedin Table.III.

$$\begin{bmatrix} X_{cs0} \\ X_{cs1} \\ X_{cs2} \\ X_{cs3} \\ \vdots \\ X_{cs(m-1)} \end{bmatrix}=\frac{1}{m}\begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & a & a^2 & a^3 & \cdots & a^{(m-1)} \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix}\begin{bmatrix} \sum_{\nu=0}^{+\infty} X_{(2\upsilon+1)} \\ \sum_{\nu=0}^{+\infty} X_{(2\upsilon+1)} e^{(2\nu+1)\frac{2\pi}{m}} \\ \sum_{\nu=0}^{+\infty} X_{(2\upsilon+1)} e^{(2\nu+1)\frac{4\pi}{m}} \\ \vdots \\ \sum_{\nu=0}^{+\infty} X_{(2\upsilon+1)} e^{(2\nu+1)\frac{(m-1)2\pi}{m}} \end{bmatrix} \quad (22)$$

The classes of harmonics of the five-phase, six-phase, seven-phase and nine-phase systems are given in Table.III, and the symmetrical multiphase which represent the sequences $X_{cs1}\ldots X_{cs(m-1)}$ and the associated harmonics are presented in Fig. 8.

C. Instantaneous Symmetric Components

Mathematically, it is possible to apply the transformation into symmetrical components on the instantaneous quantities of phases, this by using the transformation matrices C_s, C_{s2} or C_{su}. This results in instantaneous transformed quantities known as instantaneous symmetrical components (ISC) [32-33]. The ISC concept has been introduced in [30], as referenced in [29].

The ISC transformation can be expressed by (23) if the number of phases is odd, by (24) if the number of phases is even, and by equation (25) for the unitary ISC transformation.

$$\begin{bmatrix} x_{sc0}(t) \\ x_{sc1}(t) \\ x_{sc2}(t) \\ \vdots \\ x_{sc(m-1)}(t) \end{bmatrix}=\frac{1}{m}\begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & a & a^2 & a^3 & \cdots & a^{(m-1)} \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_m(t) \end{bmatrix} \quad (23)$$

$$\begin{bmatrix} x_{cs0}(t) \\ x_{cs00}(t) \\ x_{cs1}(t) \\ x_{cs2}(t) \\ \vdots \\ x_{cs(m-2)}(t) \end{bmatrix}=\frac{1}{m}\begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & 1 & -1 & \cdots & -1 \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_m(t) \end{bmatrix} \quad (24)$$

The instantaneous quantities of the phases are presented in equation (26). The ISC are presented in equation (27) for an odd number of phases and in equation (28) for an even number of phases.

Since the phase quantities—currents, voltages, fluxes, etc.—are real values, the zero-sequence component is real. In contrast, the other components are complex quantities that vary as a function of time.

$$\begin{bmatrix} x_{sc0}(t) \\ x_{sc1}(t) \\ x_{sc2}(t) \\ \vdots \\ x_{sc(m-1)}(t) \end{bmatrix} = \frac{1}{\sqrt{m}} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & a & a^2 & a^3 & \cdots & a^{(m-1)} \\ 1 & a^2 & a^4 & a^6 & \cdots & a^{2(m-1)} \\ 1 & a^3 & a^6 & a^9 & \cdots & a^{3(m-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & a^{(m-1)} & a^{2(m-1)} & a^{3(m-1)} & \cdots & a^{(m-1)(m-1)} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_m(t) \end{bmatrix} \quad (25)$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_m(t) \end{bmatrix} \quad (26)$$

$$x_{sc}(t) = \begin{bmatrix} x_{sc1}(t) \\ x_{sc2}(t) \\ x_{sc3}(t) \\ \vdots \\ x_{scm}(t) \end{bmatrix} \quad (27)$$

$$x_{sc}(t) = \begin{bmatrix} x_{cs0}(t) \\ x_{cs00}(t) \\ x_{cs1}(t) \\ x_{cs2}(t) \\ \vdots \\ x_{cs(m-2)}(t) \end{bmatrix} \quad (28)$$

The transformation presented in equation (20), which involves Fourier series development, allows representing each phase quantity by its harmonic series. This can be utilized in ISC transformation. The Fourier series development of a phase quantity is expressed in equation (29). Therefore, the phase quantities can be represented by equation (30).

$$x_k(t) = \sum_{\upsilon=1}^{+\infty} X_\upsilon \cos\left(\nu.\omega.t + \frac{(k-1)2.\pi}{m} + \phi_{k.\upsilon} \right) \quad (29)$$

$$x(t) = \begin{bmatrix} \sum_{\nu=0}^{+\infty} X_{(2\nu+1)} \cos\left((2\nu+1).\omega.t + \phi_{k.(2\nu+1)}\right) \\ \sum_{\nu=0}^{+\infty} X_{(2\nu+1)} \cos\left((2\nu+1).\omega.t + \frac{2.\pi}{m} + \phi_{k.(2\nu+1)} \right) \\ \sum_{\nu=0}^{+\infty} X_{(2\nu+1)} \cos\left((2\nu+1).\omega.t + \frac{4.\pi}{m} + \phi_{k.(2\nu+1)} \right) \\ \vdots \\ \sum_{\nu=0}^{+\infty} X_{(2\nu+1)} \cos\left((2\nu+1).\omega.t + \frac{(m-1)2.\pi}{m} + \phi_{k.(2\nu+1)} \right) \end{bmatrix} \quad (30)$$

V. Conclusion

This paper discusses the differences between multiphase systems, resulting from phase numbers characteristics. The most important conclusion is that MES is modifiable if its number of phases is factorizable. Where the number of phases can be reduced or the MES can be transformed into multi-star MES. In addition, the possible phase connections are presented, and the impact of the factorizability of the number of phases on these connections is discussed. The variation in voltages between adjacent phases is analyzed as a function of the number of phases. It is observed that this voltage decreases and approaches values lower than the phase voltage. In contrast, the voltages between non-adjacent phases increase with the number of phases and approach twice the phase voltage. The application of the symmetrical components method to multiphase systems (MPSs) is also examined, particularly in terms of how the number of phases affects the symmetrical components. For instance, the emergence of pseudo zero-sequence components in systems with an even number.

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Abdelhakim Khelafi received the B.Sc, M.Sc. and PhD degree in electrical engineering from Ecole Nationale Polytechnique (ENP), Algiers, Algeria. His research interests include multiphase machine modeling, multiphase machine design, and multiphase drives.

Abdelmalek Djebli received the B.Sc, M.Sc. and PhD degree in electrical engineering from Ecole Nationale Polytechnique (ENP), Algiers, Algeria. His research interests include multiphase machine modeling, multiphase machine design, and multiphase drives.

Omar Touhami was born in Algeria. He received the Engineer, Magister and Doctorate degrees in Electrical Engineering in 1981, 1986 and 1994 respectively from Ecole Nationale Polytechnique (ENP), Algiers, Algeria. He is currently a professor in Electrical Engineering Department at National Polytechnic School of Algiers. Its field of interest is Domain of research Electrical machines and Variable speed drives.

Rachid Ibtiouen received the Ph.D. degree in electrical engineering from Ecole Nationale Polytechnique (ENP), Algiers, Algeria, and the Institut National Polytechnique de Lorraine, Nancy, France, in 1993. He was with the Groupe de Recherche en Électrotechnique et Électronique de Nancy, Nancy, from 1988 to 1993. From 2005 to 2013, he was the Director of the Laboratoire de Recherche en Electrotechnique with ENP. He is currently a Professor and the Associate Director of research with ENP. His current research interests include the modeling of electric systems and drives, and, in particular, electrical machines. Dr. Ibtiouen was an Expert Member of the Committee of Evaluation of the University Projects of Research with the Comité National d’Evaluation et de Programmation de la Recherche Universitaire, Ministry of the Algerian Higher Education, from 1993 to 2011.