Stochastic Analysis to Predict Reliability Index of a Tall Building Structure

Badreddine Chemali and Boualem Tiliouine

Abstract− This article presents a stochastic analysis to assess the probability of failure and reliability of tall building structures with random variables under stochastic seismic excitation, using the conventional Monte Carlo Simulation (MCS) method. Uncertainties in seismic loading, structural geometry, and material characteristics are incorporated t in the study. Furthermore, the sensitivity of structural reliability is examined in relation to different performance variable limit states. The numerical results demonstrate that structural reliability is significantly influenced by the variability of all random variables but more importantly by seismic loading randomness. It's shown that, when the variability of the random parameters is higher, the effects on structural reliability are more noticeable. In addition, preliminary sensitivity analysis based on the First Order Reliability Method (FORM) that gives information on the sensitivity of the randomness of the inputs parameters, shows that the 11 stochastic input parameters seismic probabilistic problem can be effectively reduced utilizing only 4 random variables, namely: the Peak Ground Acceleration, concrete elastic modulus, core inertia and reinforced concrete density.

***Keywords*− First Order Reliability Method, MCS, Peak Ground Acceleration, sensitivity, tall building structures.**

NOMENCLATURE

PGA	Peak Ground Acceleration
MCS	Monte Carlo Simulation.
FORM	First Order Reliability Method.
$\overline{X}$	Mean value of random variable
P_f	Probability of failure
R	Reliability.
$g(X,\overline{X})$	Performance function.
Φ	Cumulative Standard Normal Distribution Function.
β_{HL}	Hasofer-Lind Reliability index
Α	sensitivity factor
RPA 2024	Algerian Earthquake Regulation
A	Zone acceleration coefficient
η	correction factor of damping
ξ	percentage of critical damping
$\bar{y}$	allowable maximum displacement
ρ	Density
I	Inertia
E	elastic modulus

I. INTRODUCTION

Uncertainties in the construction process may be categorized into two primary types: natural variability and human-induced factors (e.g. [1]). Natural uncertainties stem from unpredictable environmental loads (wind, seismic, snow, live loads) and material behavior variations, while human-induced uncertainties arise from design approximations, computational errors, etc.

Manuscript received October 3, 2023; revised February 22, 2024; revised July 7, 2025.

Badreddine Chemali and Boualem Tiliouine are with the Civil Engineering Department, Ecole Nationale Polytechnique, Algiers, ALGERIA (e-mail: badar093@yahoo.fr; boualem.tiliouine@g.enp.edu.dz).

Digital Object Identifier (DOI): 10.53907/enpesj.v5i2.244

The evolution of computational power has enabled the explicit incorporation of uncertainty quantification in structural analysis, facilitating deeper understanding of the behavior of probabilistic structures. (e.g. [2-5])

This study investigates the effectiveness of conventional Monte Carlo Simulation (MCS) in assessing structural performance and system reliability for high-rise buildings. Failure is defined as the exceedance of the building's lateral top displacement beyond code-specified limits (e.g., H/500 as per IBC design code [6]). Stochastic analyses are conducted on a representative high-rise buildings under seismic loading, with the following probabilistic modeling:

- Geometrical parameters as independent normal random variables
- Structural materials and loads modeled as lognormally distributed

The sensitivity of structural reliability to performance criteria is evaluated by varying the limit-state thresholds. Furthermore, the capabilities and limitations of the MCS approach are critically examined. In addition, preliminary sensitivity analysis based on FORM is conducted to identify critical input parameters, followed by interpretation of practical engineering consequences.

The paper is organized as follows: Reliability analysis methods are briefly described in Section 2. A description of Tall Building Structure Example is provided Section 3. The overall results of stochastic analyses to predict reliability index of a tall building structure with uncertain parameters under random seismic loading are presented in Section 4. Finally, the study concludes with a synthesis of key findings and their implications for structural reliability analysis.

II. SOME BACKGROUND ON RELIABILITY ANALYSIS METHODS

In the conventional probabilistic framework, the uncertainties are modeled as random parameters with certain distribution characteristics. Let denote X the vector of uncertain input parameters and $\overline{\mathbf{X}} = [\overline{X}_1,...,\overline{X}_m]^T$ the vector of deterministic input parameter (mean values of uncertain variables); the probability of failure Pf can be given as:

$$P_f = P\left[g(\mathbf{X},\overline{\mathbf{X}}) \le 0\right] = \int \ldots \int_{g(\mathbf{X},\overline{\mathbf{X}})\le 0} p_{\overline{X}}(\mathbf{x},\overline{\mathbf{x}})d\overline{\mathbf{x}}_1..d\overline{\mathbf{x}}_m \quad (1)$$

where $g(\mathbf{X},\overline{\mathbf{X}}) = \bar{y} - y(\mathbf{X},\overline{\mathbf{X}})$ is the limit state function and $g(\mathbf{X},\overline{\mathbf{X}}) \le 0$ expresses the failure event, hence the reliability can be defined as $g(\mathbf{X},\overline{\mathbf{X}}) > 0$; $\overline{\mathbf{x}} = [\overline{\mathbf{x}}_1,...,\overline{\mathbf{x}}_m]^T$ represents the realization of $\overline{\mathbf{X}}$, $\mathbf{p}_{\overline{\mathbf{x}}}(\mathbf{x},\overline{\mathbf{x}})$ represents the joint probability density function of the system parameters, typically estimated from empirical measurement data. The variable y denotes the performance metric, while $\bar{y}$ corresponds to its critical threshold (limit state).

For evaluating the probability of failure described in equation 1, various numerical methods are available for this purpose, including Monte Carlo Simulation (e.g. [3,7]), First- and Second-Order Reliability Methods (FORM/SORM) (e.g.[1]), Point Estimate Methods (e.g. [8]) and Response Surface Method (e.g. [9]).

A measure of the system reliability can be given by the Reliability Index

$$\beta = \Phi^{-1}(1 - P_f) \quad (2)$$

where Φ^{-1} denotes the inverse standard normal cumulative distribution function. The reliability R is computed by:

$$R=\Phi(\beta) \quad (3)$$

where Φ represents the standard normal cumulative distribution function

II.1 Monte Carlo Simulation technique

In stochastic assessment, the MCS method is habitually utilized when the analytical solution is not possible and the failure domain cannot be described by an analytical form. The Monte Carlo Simulation (MCS) method is particularly indispensable for complex problems involving numerous stochastic input parameters, where conventional reliability analysis techniques (e.g., FORM/SORM) prove inadequate. While MCS boasts a straightforward mathematical formulation and unparalleled versatility in handling problems of arbitrary complexity, its principal drawback lies in the prohibitive computational cost associated with traditional implementations—especially for high-dimensional or low-probability failure events. One basic advantage of the Monte Carlo Simulation over the other reliability analysis methods for the particular type of problems investigated in the present study is that its efficiency is not affected by the additional complexities due to non-linear analysis and the dynamic loads.

The computational cost of MCS grows proportionally when the number of input parameters is large or/and the magnitude of P_f is small, since both cases require a huge sample size. For this reason, various sampling techniques, also called variance reduction techniques, have been developed in order to improve the computational efficiency of the method in order to minimize the sample size and minimize the statistical inaccuracy that is intrinsic to MCS approaches.

Expressing the limit state function as $g(\mathbf{X},\overline{\mathbf{X}}) \le 0$, where $\overline{\mathbf{X}} = [\overline{X}_1,...,\overline{X}_m]^T$ is the vector of the random variables and since MCS is based on the theory of large numbers (N∞), an unbiased estimator of the probability of failure is given by

$$P_f = \frac{1}{N_\infty}\sum_{j=1}^{N_\infty} I(\mathbf{x}_j) \quad (4)$$

in which, $I(x_j)$ is an indicator for successful and unsuccessful simulations defined as

$$I(\mathbf{x}_j) = \begin{cases} 1 & \text{if} \quad g(\mathbf{X},\overline{\mathbf{X}}) \ge 0 \\ 0 & \text{if} \quad g(\mathbf{X},\overline{\mathbf{X}}) < 0 \end{cases} \quad (5)$$

To estimate the failure probability P_f, the Monte Carlo method generates N independent random realizations of the input vector **x**, typically sampled from a uniform probability distribution. For each realization x_j, the limit-state function $g(\mathbf{x}_j)$ is evaluated. The failure probability is then estimated as:

$$P_f = \frac{n}{N} \quad (6)$$

where N represents the total number of Monte Carlo trials and n denotes the count of simulations where the response exceeds the deterministic case (evaluated at mean input values). Traditional MCS is that in order to acquire an accurate prediction of output first and second moments, the analysis may require thousands of simulation runs, resulting in computationally intensive and resource-demanding calculations.

To enhance computational efficiency while preserving the accuracy of Monte Carlo Simulation, several variance reduction techniques have been developed (e.g. [1, 5, 10-11]).

III. Description of Tall Building Structure Example

The case study examines a 35-story wall-frame structure (height: 122.5 m) as shown in Figure 1 [12]. The lateral load-resisting system for seismic actions along the long facade consists of:

- Six moment-resisting frames
- A central core ($I = 313$ m⁴)

With a concrete elastic modulus (E) of 2×10^7 kN/m², this analysis aims to quantify the structural reliability under seismic loading conditions compatible with the design spectra derived from Algerian Earthquake Regulations [13] based on five parameters: soil types S1 (rocky site), category 1A with an Importance coefficient I =1.2. The structure is assumed to be located in a moderate seismicity zone (zone II a) characterized by an acceleration coefficient A = 0.25g (see Fig. 2). Table I presents the moment of inertia values for frame columns and girders, along with their statistical distributions, for the 35-story case study building.

The seismic action is represented by the following Design Response Spectrum as per the Algerian Earthquake Regulations (RPA2024)

A: Zone acceleration coefficient

η: damping correction factor

$$\eta = \sqrt{\frac{7}{2+\xi}} \quad \text{(used when } \xi \text{ is not equal to 5\%)}$$

ξ: damping ratio

I: Importance coefficient

T1, T2: characteristic site periods corresponding to the designated soil category

S: Site coefficient

$$\frac{S_a}{g} = \begin{cases} A.I.S\left(1+\frac{T}{T_1}(2.5\eta-1)\right) \ldots\ldots\ldots\ldots\ldots\ldots 0 \le T \le T_1 \\ A.I.S(2.5\eta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots T \le T \le T_2 \\ A.I.S(2.5\eta)\left(\frac{T_2}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots T_2 \le T \le T_3 \\ A.I.S(2.5\eta)\left(\frac{T_2T_3}{T^2}\right) \ldots\ldots\ldots\ldots\ldots T_3 \le T \le 4s \end{cases} \tag{7}$$

The top displacement is selected as the governing parameter for the performance function evaluation.

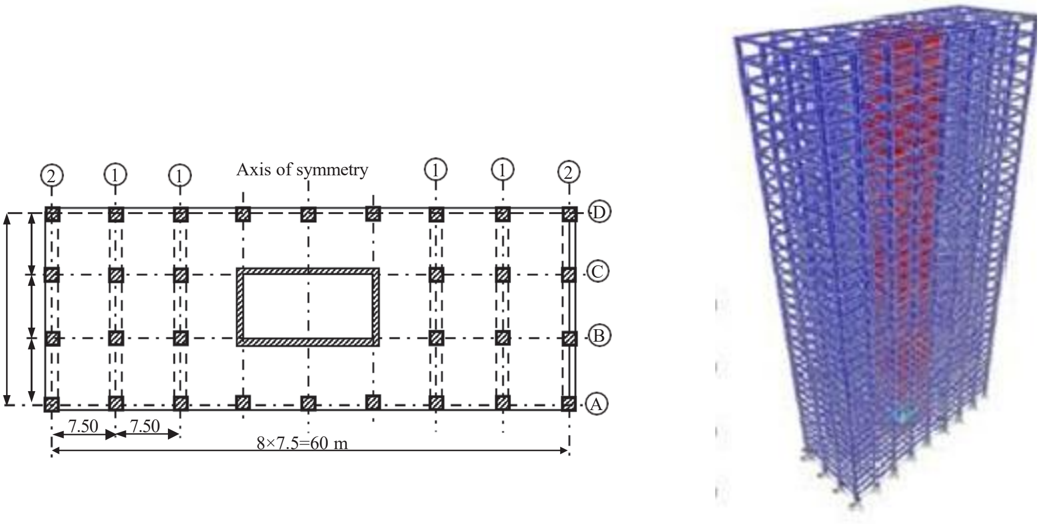
As quantified in Table I, the stochastic input variables comprise:

- Geometric parameters (e.g., member dimensions),
- Material properties (e.g., concrete strength), and
- Seismic loading, characterized through Peak Ground Acceleration (PGA) variability

It should be noted that there is Additional stochastic parameters significantly influence structural reliability assessments, particularly live load uncertainties (e.g. [14]), spatially correlated soil variations (e.g. [15]), and soil-structure interaction complexities (e.g. [16]) etc...For the sake of clarity we will focus on the variables mentioned above.
The corresponding limit state function takes the form:

$$g(I_{ci}, I_{bi}, \ldots, PGA) = \bar{y} - y(\mathbf{z} = \mathbf{H}) \tag{8}$$

The limit state compares the allowable top displacement ($\bar{y}$) against the response spectrum-derived demand. A parametric reliability analysis was conducted by progressively increasing $\bar{y}$ and computing the failure probability via MCS at each step. The study employed 80,000 stochastic samples of PGA and first-mode frequency, with their distributions shown in Figures 3–4.



(a) (b)

Fig. 1. Plan (a) and 3-D view (b) of 35-story wall-frame

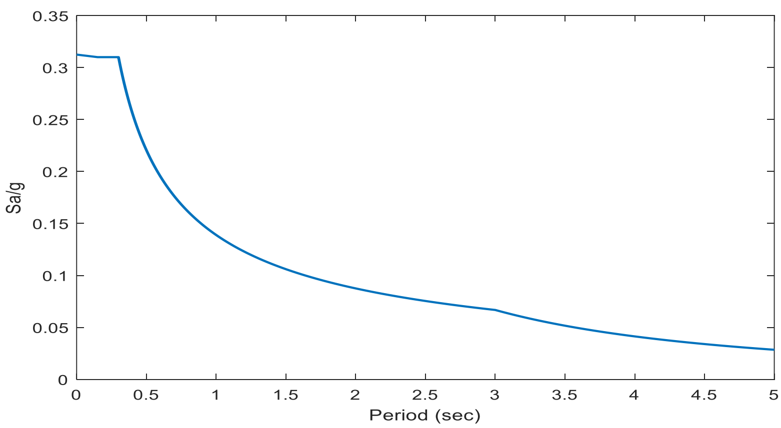


Fig. 2. Elastic design response spectrum

TABLE I
STRUCTURAL PARAMETERS AND STATISTICAL DATA FOR 35-STORY HIGH-RISE BUILDING

Parameters	Stochastic variable	Symbol	Mean	Coefficient of variation	Distribution
	Core	I_{cor}	313 m^4		
	Frame 1	I_{ic1}	0.083 m^4		
		I_{ec1}	0.050 m^4		
geometrical		I_{g1}	0.011 m^4	0.05	Normal
	Frame 2	I_{ic2}	0.050 m^4		
		I_{ec2}	0.034 m^4		
		I_{g2}	0.005 m^4		
	Elastic modulus	E	2 x 10^7 KN/m^2	0.15	
material	Damping Ratio	ξ	7%	0.25	Lognormal
	Density	P	2500 Kg/m^3	0.20	
Loading	Peak Ground Acceleration	PGA	0.25g	0.30	Lognormal

Where i: Interior, E: exterior, c: column and g: girder.

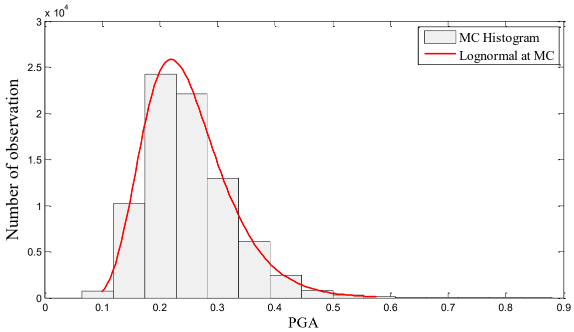


Fig. 3. Histogram for PGA generated with MCS technique.

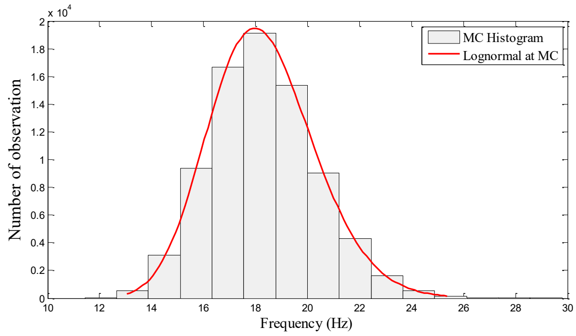


Fig. 4. Histogram for frequency of first vibration mode of 35 story building, generated with MCS

IV. RESULTS AND DISCUSSION

A comprehensive evaluation of the proposed structural reliability methodology's practical utility and performance characteristics is presented in this section. The case study structure enables an efficient numerical solution through modal superposition techniques (e.g. [17]), significantly enhancing the computational feasibility of Monte Carlo Simulation (MCS) implementation.

Through combined seismic response spectrum analysis and conventional Monte Carlo Simulation (MCS), the structural reliability assessment yields:

- Probability of failure (P_f) = 4.94 × 10^{-2} (95.06% reliability)
- Reliability index (β) = 1.651

These MCS-derived results are comprehensively summarized in Table II.

TABLE II.

RELIABILITY RESULTS of Tall Building Structures MCS

y (mm)	P_f	R (%)	βMCS
10	100	0,000	-8,112
50	99,931	0,069	-3,200
125	65,654	34,346	-0,403
143	49,698	50,302	0,008
145	48,008	51,992	0,050
175	26,634	73,366	0,624
245 (H/500)	4,938	95,062	**1,651**
300	1,163	98,837	2,269
350	0,308	99,692	2,739
400	0,082	99,918	3,147

For all the points $\sigma_y = 51.8\,mm$

The probability distributions of both seismic intensity (PGA) and structural reliability (performance variable CDF) are shown in Figures 5 and 6 respectively, with their theoretical PDF fits. These results were obtained through extensive Monte Carlo simulation (N = 80,000 realizations).

Figure 7 presents the computed failure probabilities for the case study structure as a function of the seismic load Coefficient of Variation (COV). The probabilities correspond to exceedance of the specified limit state for top displacement (the selected performance variable). The results demonstrate a clear positive correlation between failure probability and increasing seismic load variability.

The First-Order Reliability Method (FORM) offers an additional valuable feature through its directional cosines [1], which quantify the sensitivity of the reliability index to each random input variable. This sensitivity information is particularly important for robust design optimization. A preliminary FORM-based sensitivity analysis was performed, revealing that variables with sensitivity measures below a specified threshold α (Equation 9) could be treated as deterministic. For this case study, α was set at 3%, allowing identification of parameters with negligible influence on the structural reliability. It should be mentioned that although the First-Order Reliability Method (FORM) offers directional cosines for sensitivity ranking, other reliable methods, like the response surface method (e.g. [18]) or incomplete Monte Carlo simulation (e.g. [19]), provide complementary benefits for sensitivity analysis using reliability index.

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial x_i'}\right)^* \sigma_{x_i'}}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial g}{\partial x_i'}\sigma_{x_i'}\right)^{2*}}} \qquad (9)$$

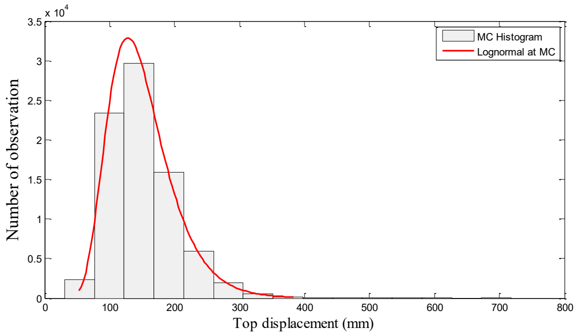


Fig. 5. Histograms for top displacement generated with MCS

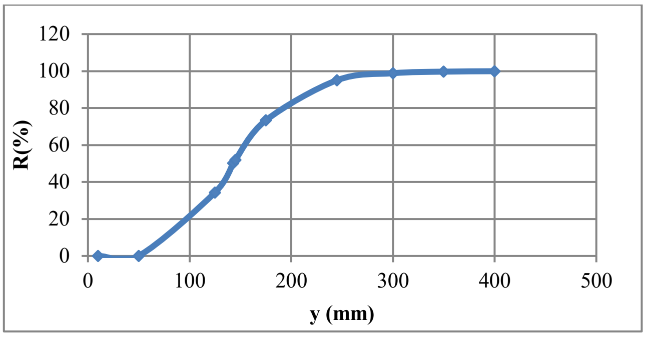


Fig. 6. Reliability of study tall building structure calculated by MCS

The reliability analysis was thus simplified from an 11-dimensional random variable problem to a 4-variable formulation, retaining only the most influential parameters:

1. Peak Ground Acceleration (PGA, α = 0.861)
2. Concrete elastic modulus (E, α = 0.291)
3. Core moment of inertia (I, α = 0.095)
4. Reinforced concrete density (ρ, α = 0.379)

Figure 8 demonstrates the characteristic convergence behavior of the failure probability estimate as sample size increases. While Monte Carlo Simulation (MCS) with N realizations provides robust reliability estimates, its computational demand becomes significant for systems with numerous degrees of freedom (DOFs). For this case study, the analysis employed $8x10^4$ simulations, determined through progressive assessment of failure probability convergence versus sample size. More complex scenarios may require substantially larger sample sizes due to slower statistical convergence. In the current implementation (11 random input variables and one output performance function), the MCS required 9 minutes and 23 seconds of CPU time for 8 $x10^4$ samples.

V. CONCLUSIONS

The design of tall building structures is inherently influenced by multiple uncertainty sources. However, through systematic reliability-based approaches, structural safety can be enhanced to meet or exceed codified reliability thresholds.

The analysis demonstrates that structural reliability is sensitive to variability in all uncertain parameters, with particularly strong dependence on loading randomness, density of concrete, core inertia and concrete elastic modulus uncertainty. Furthermore, the effects on structural reliability have been shown to be more pronounced for higher variability of the stochastic variables.

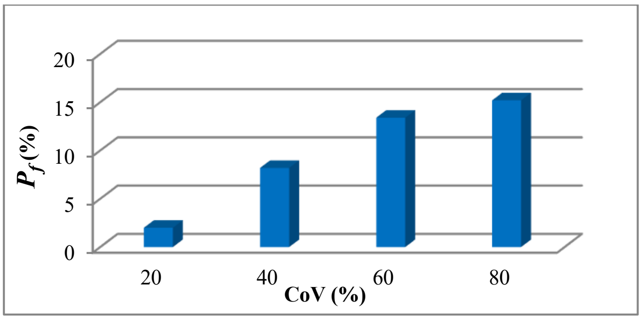


Fig. 7 Structural Failure Probability as Function of Seismic Load COV

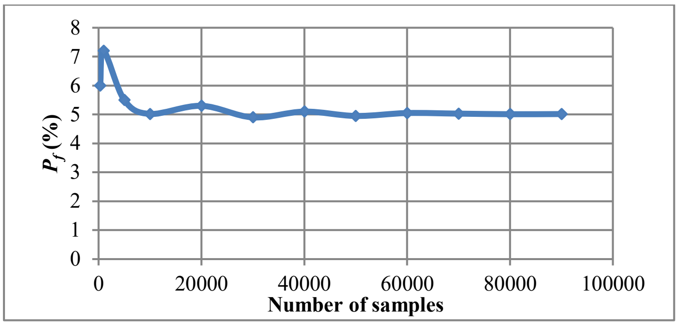


Figure 8 Convergence of probability of failure with increasing sample size

Monte Carlo Simulation (MCS) for stochastic analysis of tall buildings often requires substantial computational resources, particularly for structures with numerous degrees of freedom and multiple random variables. Implementing variance reduction techniques combined with sensitivity analysis via directional cosines can significantly improve computational efficiency while maintaining accuracy - a crucial consideration for robust design applications.

REFERENCES

[1] Farsangi, E. N., Noori, M., Gardoni, P., Takewaki, I., Varum, H., & Bogdanovic, A. (Eds.). “Reliability-based analysis and design of structures and infrastructure”. CRC Press. 2021.

[2] Y. Tsompanakis, V. Papadopoulos, N. D. Lagaros and M. Papadrakakis, “Reliability analysis of structures under seismic loading”. Fifth World Congress on Computational Mechanics. Vienna, Austria, July 7-12, 2002

[3] B. Chemali and B. Tilioiune,. “Uncertainty propagation in dynamics of structures with correlated damping using a nonlinear statistical model” Int. J. Struct. Eng , 7 (2), pp 145 – 159. 2016. https://doi.org/10.1504/IJSTRUCTE.2016.076692

[4] Aldosary, M., Wang, J., & Li, C. . “Structural reliability and stochastic finite element methods: State-of-the-art review and evidence-based comparison”. Engineering Computations, 35(6), 2165-2214. 2018 https://doi.org/10.1108/EC-04-2018-0157

[5] Teng, D., Feng, Y. W., Chen, J. Y., & Lu, C. “Structural dynamic reliability analysis: review and prospects. International Journal of Structural Integrity”, 13(5), 753-783. 2022 https://doi.org/10.1108/IJSI-04-2022-0050

[6] International Code Council, International Building Code. Whittier, California, USA. 2009

[7] M.H. Kalos and P.A. Whitlock, “Monte Carlo Methods”, 2nd ed New York: Wiley. 2002

[8] E. Rosenblueth, “Point estimates for probability moments”. Proc., Nat. Acad. of Sci., vol. 72 no. 10, pp. 3812–3814 , 1975. https://doi.org/10.1073/pnas.72.10.3812

[9] Chemali, B., & Tilioune, B. (2023). Probabilistic analysis of shallow foundations on c-φ soils using 2nd order response surface methods. Periodica Polytechnica Civil Engineering, 67(2), 485-494. https://doi.org/10.3311/PPci.17917

[10] Pulido, T.L. Jacobs, E.C. Prates De Lima, “Structural reliability using Monte-Carlo simulation with variance reduction techniques on elastic-plastic structures”, Comp. & Struct. vol. 43, pp. 419-430. 1992. https://doi.org/10.1016/0045-7949(92)90275-5

[11] Y. Cao, M.Y. Hussaini, and T.A. Zang, “Exploitation of sensitivity derivatives for improving sampling methods” , AIAA Journal. vol. 42, no. 4, pp. 815‑822, 2004. https://doi.org/10.2514/1.2820

[12] Y Ohtori, RE Christenson, BF Spencer Jr, SJ Dyke, Benchmark Control Problems for Seismically Excited Nonlinear Buildings, J. Eng. Mech. 130, 366 (2004)

[13] Ministère de l’Habitat, Document Technique Réglementaire DTR B C 2 48, Règles Parasismiques Algériennes RPA 2024 Centre National de Recherche Appliquée en Génie-Parasismique, 2024.

[14] Costa, L. G., & Beck, A. T. (2024). A critical review of probabilistic live load models for buildings: Models, surveys, Eurocode statistics and reliability-based calibration. Structural Safety, 106. 2024.
https://doi.org/10.1016/j.strusafe.2023.102411

[15] Luo, Z., Kim, M., & Hwang, S. (2021). Effect of soil spatial variability on the structural reliability of a statically indeterminate frame. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 7(1), 04020048.2021
https://doi.org/10.1061/AJRUA6.0001098

[16] Bezih, K., Chateauneuf, A., Kalla, M., & Bacconnet, C. Effect of soil–structure interaction on the reliability of reinforced concrete bridges. Ain Shams Engineering Journal, 6(3), 755-766. 2015.
https://doi.org/10.1016/j.asej.2015.01.007

[17] Chopra, A. K. “Dynamics of structures”. 4th ed. Pearson, USA. 2022.

[18] Zhou, C., Li, C., Zhang, H., Zhao, H., & Zhou, C. (2021). Reliability and sensitivity analysis of composite structures by an adaptive Kriging based approach. Composite Structures, 278, 114682.
https://doi.org/10.1016/j.compstruct.2021.114682

[19] Tandjiria, V., Teh, C. I., & Low, B. K. (2000). Reliability analysis of laterally loaded piles using response surface methods. Structural safety, 22(4), 335-355.
https://doi.org/10.1016/S0167-4730(00)00019-9

Badreddine Chemali received his Msc (Magister) and Ph.D. in Civil Engineering from École Nationale Polytechnique (ENP), Algiers in 2024. He holds membership in the Earthquake Engineering and Structural Dynamics Laboratory (LGSD) within ENP's Civil Engineering Department. Currently, Dr. Chemali serves as a Reservoir Engineering Specialist in the Production Division of SONATRACH, Algeria's national oil and Gas Company.

Boualem Tiliouine received his Msc and PhD degrees in Civil Engineering from Stanford U., USA and is a former Research Fellow at the Blume’s Earthquake Engineering Center of Stanford University. He is currently working as a full time Professor at the “Ecole Nationale Polytechnique” (ENP), Algiers. He has published over 120 papers in reputed international journals and conferences. He has been chief investigator of several nationally and internationally funded research projects and has over 30 years of experience in teaching, research and consultancy. His previous positions and work experience include General Director of ENP, Structural Engineering Expert and Technical Adviser at CE Department of Dubai Municipality. His research interests include earthquake engineering, structural dynamics, soil structure interaction and optimization. He is Honorary Fellow of the National Academic Committee of the Ministry of Higher Education and Scientific Research (Algeria) and active member of the French Association of Earthquake Engineering.