Towards Optimal Frequency Plans: A Survey of Frequency Assignment Strategies, Models, and Methods

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Abstract−The frequency assignment problem in a telecommunication network refers to the task of assigning frequencies to transmitters to minimize or eliminate interference and to optimize at least one other parameter. The literature on this task is vast and examines many parameter optimization objectives. Numerous schemes, models, and methods have been proposed. In this article, we present an extensive survey of the published studies on the frequency assignment problem in the context of a wide range of applications. Different types of optimization objectives, models and solution methods are presented in a systematic way.

***Keywords*− Mobile computing; Telecommunication networks; Optimization; Heuristics and metaheuristics.**

I. INTRODUCTION

The frequency assignment problem (FAP), also called the channel assignment, frequency, spectrum sharing, or spectrum management problem, is the task of allocating communication frequencies to users without compromising the quality of network communications. This problem arises in numerous real situations that have a direct impact on the quality of modern living. In an ever-increasing share of the world, the use of mobile phones and telecommunications in general (sound, image, and data) has become crucial for individuals and businesses to thrive. This may be considered one of the most significant socioeconomic advances in recent decades. Figure 1 depicts the evolution of telecommunications over time from 2020 and a forecast to 2025. The number of phone users and network service providers is growing steadily.

According to International Telecommunications Union subscription statistics, 6.42 billion mobile phones were in circulation in 2022 [1]. In 2017, Statista projected the number of mobile devices to reach 18.22 billion by 2025, which would be an increase of 4.2 billion from the count for 2020 [2]. While the numbers continue to grow, the spectrum of frequencies available to accommodate all these users remains the same.

Subject to certain constraints, frequencies are natural resources that can be shared and reused to manage the quality, coverage, and cost of telecommunication network services.

Manuscript received November 29, 2023; revised December 19, 2023.
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Digital Object Identifier (DOI): 10.53907/enpesj.v3i2.249

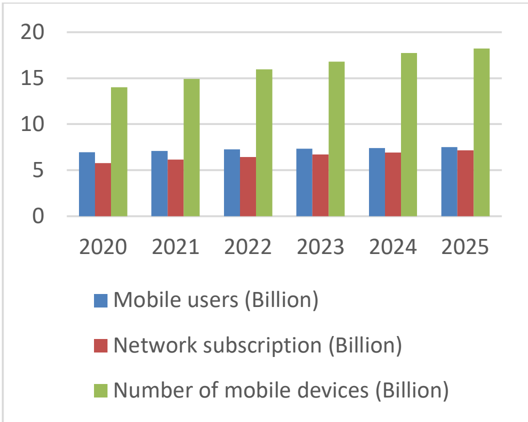

Fig. 1: Number of mobile devices, users, and network subscriptions evolution over time (image generated using data in [1–3]).

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The FAP was first addressed in its simplest form in 1977 [4] but became more and more complex with the growth in the number of users and technological advances in the telecommunications field [5]. The frequency assignment problem has since been proven to be an NP-hard problem [6]. Many models, algorithms, and theories have been introduced and studied in search of lasting solutions.

Rather than simply list all previous articles published in this subject area, we seek to propose a functional directory. No survey has yet provided a complete overview of the field. Most published surveys have focused on a specific category of FAPs or a specific application and formulation, and they do not give readers a comprehensive under-standing of the problem overall, the research trends, or recently proposed solutions.

II. METHODOLOGY

To perform the literature review, works from the last twenty years were considered in addition to a few older references that are still important to the field. The works were mainly journal articles on the topic of modelling and solving the FAP that are listed in the SCOPUS database and were published in highly ranked IEEE, Elsevier, or Springer journals. Related topic conference papers were also considered from well-reputed conferences such as IEEE conferences and computer science related conferences.

The search words used to find related works were “frequency assignment problem” and “channel assignment problem” associated with one or more of the following words: modelling, optimization, solution, solving, methods, telecommunication, net-work, and wireless networks. Only papers dealing directly with the FAP and studies expanding on methodologies found in the literature were evaluated. Papers that presented generalities about the issue without mathematical breakthroughs or practical examples were not considered.

The papers were categorized according to the type of FAP, the modelling method, and the optimization method, as specified in Figure 2.

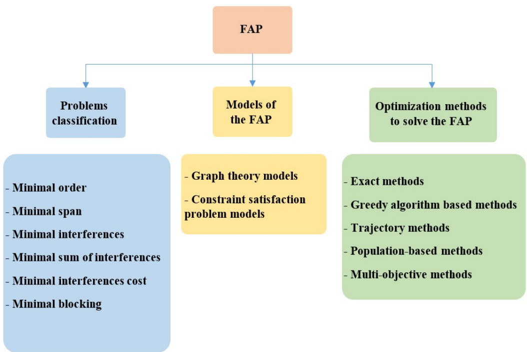

Fig. 2: Classifications of FAP problems, models, and optimization techniques found in the literature.

This article is structured according to Figure 3. Following a survey of FAP schemes and classes (Section 4), models (Section 5), and methods (Section 6), we propose a library of concepts to increase the effectiveness of FAP studies.

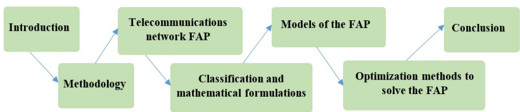

Fig. 3: The structure of the review.

III. TELECOMMUNICATION NETWORK FREQUENCY ASSIGNMENT PROBLEM S

A telecommunication network comprises a set of transmitters distributed over a set of utilization sites. The transmitters communicate with each other and with terminals (mobile phones and other devices) through radio links carried on frequencies or channels. Frequencies are scarce natural resources. The word scarce in this context means that, although they are reusable, the availability of frequencies is finite. All telecommunication networks must therefore share them in order to satisfy all the requests of the users and providers. However, the reuse of frequencies may cause deterioration of received signal quality. This deterioration may be mitigated by a suitable frequency assignment. A provider wishing to establish a network must implement a frequency plan capable of supporting the required coverage and service quality. In a functional network, the frequency assignment is updated continually as new requests are accommodated and current communications end.

The FAP arises at both the design and the deployment stages in all telecommunication networks, including mobile telephony, TV and radio broadcasting, military and other wireless communication systems, the Internet of Things, and cognitive networks [5–11]. In its simplest and most generalized form, the assignment problem is equivalent to the graph colouring problem, which is a well-known NP-hard problem [5–9].

In the operations research literature, FAP in telecommunication networks has attracted much attention because of its practical relevance and close relationship with classical combinatorial optimization problems. Our aim here is to present the concepts and ideas that are common to all applications rather than focus on mobile telephone networks. We therefore use the word ‘network’ to specify any telecommunication network, including satellite and TV networks. Similarly, the word ‘transmitter’ may designate a satellite, a base station, or any other device with similar functionality.

A. Basic concepts

To guarantee the quality of network communications, the signals within and between all radio links or other connections must not deteriorate. Loss of signal is called interference and depends mainly on geographical distance, obstacles, hardware characteristics and configuration, climate, and other largely unavoidable factors [12]. Similar frequencies or channels can be used simultaneously by several transmitters if they transmit at frequencies different enough to avoid interference or keep it below an acceptable limit. The difference between assigned frequencies must be maintained for each pair of potentially interfering communications until these communications end. These separations are called interference constraints or electromagnetic constraints. They are expressed in the following form [11, 13]:

$$|f_i - f_j| \geq s_{ij}, \quad (1)$$

where f_i and f_j are the frequencies assigned to transmitters i and j, and s_{ij} is the required separation. Three main types of separation values are recognized:

- Co-channel separation, for which the same frequency or channel cannot be assigned to certain pairs of transmitters.
- Adjacent-channel separation, for which transmitters corresponding to adjacent sites cannot be assigned adjacent channels simultaneously.
- Co-site separation, for which the frequencies assigned to the same transmitter must be separated by a certain number of frequencies.

Other constraints have been described, including duplex constraints between the two paths (both communication directions) [14, 15], and six equality and inequality constraints have been defined [16].

B. Instances definition

A FAP is defined by a 4-tuple (E, F, R, I) whose elements are as follows:

- E is the set of n network transmitters $(e_1, e_2, \ldots, e_n)$.
- F is the set of all available frequencies, defined as the union of all subsets F_i of available frequencies for each transmitter e_i $(1 \leq i \leq n)$ such that $F = F_1 \cup F_2 \cup \ldots \cup F_n$. In a small number of studies, these sets are considered as domain constraints [14].
- R is the function $R: E \rightarrow \mathbb{N}$ that defines a non-negative number of requests R_i for each transmitter e_i, and hence the

number of frequencies to assign to e_i.
Such as ℕ is the set of whole numbers.

- I is the function $I: E \times E \rightarrow R$ defining for each pair of transmitters e_i and e_j the minimal separation I_{ij} to be respected by the assigned frequencies to avoid interference (as described in III.A.), meaning that

$$\forall i, j \in [1, n], \forall p \in [1, R_i], \forall q \in [1, R_j]: \quad |f_{pi} - f_{qj}| \geq I_{ij}, \tag{2}$$

such that f_{pi} is the pth frequency assigned to the transmitter e_i.

In this general formulation, the geographical distance separating the transmitters is implicit in the function I, whereas other formulations consider the distance as its own function $E \times E \rightarrow R$, with I as a function $I: R \rightarrow R$ such that $I(d)$ is the smallest separation to be respected by the frequencies assigned to any pair of transmitters that are d units apart (to avoid interference).

Some researchers define the interferences as their level in an $n \times n$ matrix $M = \{(\mu_{ij}, \sigma_{ij})\}_{n \times n}$, in which μ_{ij} and σ_{ij} represent the mean and the standard deviation, respectively, of a Gaussian probability distribution describing the signal-to-interference ratio when the sectors in which transmitters i and j are installed and operate on the same frequency [17, 18]. The mean must be kept as high as possible to ensure adequate communication quality [19].

Other classes of FAP consider cost as a function of either distance or pairs of transmitters. The cost function C_{sig} used in GSM networks is defined for each pair of transmitter sectors relative to the frequencies assigned to them [17, 18].

The solution of a problem thus formulated is the assignment of a_i frequencies to each transmitter $e_i \in E$ $(a_i \leq R_i)$ from its nominal set F_i. Usually encoded as an assignment vector or a partition vector, this must satisfy at least some of the interference constraints defined by I and thus optimizes one or many of the objective functions of the assignment problem (considered in detail in the next section).

In assignment vector representation, transmitters are the entries, and each element is a vector containing the a_i frequencies assigned to transmitter e_i. In partition representation, the entries are the frequencies, and each element is a vector that includes the transmitters to which each frequency is assigned [8].

Coding the solution as a binary $n \times m$ matrix X, such that the cardinality of the set F $m = Card(F)$, has been suggested [12]:

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}. \tag{3}$$

Here, each element x_{ij} is defined as follows:

$$x_{ij} = \begin{cases} 1, \text{if the frequency } f_j \text{ is assigned to the transmitter } e_i, \\ 0, \text{otherwise.} \end{cases} \tag{4}$$

The following conditions must be satisfied:

$$\forall 1 \leq i \leq n, \sum_{j=1}^{m} x_{ij} = a_i \quad \text{and} \quad a_i \leq R_i. \tag{5}$$

In this coding, the relation between the assigned frequency and the corresponding fulfilled request does not appear. A new storage format for representing a solution has been proposed to reduce the space allocated to it [14]. Frequency assignment has also been addressed simultaneously with other problems such as realistic interference constraints [20], polarization [21, 22], and coverage planning via antenna localization [23].

IV. Classification and Mathematical Formulations

The FAPs studied in the literature cover a wide range of scenarios with diverse characteristics. Additionally, vendors set their own policies, which are often proprietary in-formation, making it impossible to share or compare results. This has led to the development of many types of problems and models, considering various aspects such as frequency availability, number of requests, communication between transmitters, interference management, and objectives to be optimized.

A. *Frequency assignment problem schemes*

Published frequency assignment algorithms are based mostly on one of three techniques.

(i) Fixed channel assignment

In fixed channel assignment, each transmitter is allocated a nominal set of frequencies from which frequencies to be assigned to specific requests are chosen [24]. For a given request, if no non-interfering frequency is available, as may occur in high-traffic scenarios [25], the request is blocked (rejected). The advantage of fixed channel assignment is that all transmitters act independently and no communication between them is required. However, blocking requests is an unsatisfactory solution, especially when adjacent transmitters have unused frequencies.

(ii) Borrowing-based assignment

Borrowing-based assignment is a scheme in which an unused nominal frequency assigned to a neighbour is borrowed when this does not interfere with previous assignments [26, 27].

(iii) Dynamic channel assignment

In dynamic channel assignment (DCA), all transmitters share the same set of frequencies and each assignment leads to an exchange of updates of current channel use to allow selection of a non-interfering frequency. Two dynamic channel assignment schemes are known:

- Centralized, in which a central authority is responsible for all assignments and updating the status of the entire network. The transmitters execute only the channel-use decisions made by the central base.
- Distributed, in which the network status and channel-use decision-making are distributed among the transmitters, and communications between them cannot be avoided.

No scheme is optimal in all scenarios. In all cases, the main criterion for choosing one over another is the denial ratio, that is, the proportion of rejected requests. A fixed channel scheme is adequate when the traffic is low. In especially low-traffic situations, borrowing performs better than fixed schemes. Dynamic schemes usually outperform fixed schemes in low-traffic situations but perform poorly with high traffic. This subject has been reviewed in detail [28].

B. *Classification of frequency assignment problems*

Since 1977, providers of telecommunication networks have defined several objectives and policies and hence FAP formulations. Engineers usually attempt to minimize one or more of the parameters described previously [29]. We begin here by formulating a fixed scheme in which all frequency domains for all transmitters are the same:

$$\forall i, 1 \le i \le n : F = F_1 = F_2 = \cdots = F_n. \quad (6)$$

(i) Number of assigned frequencies:

The number of assigned frequencies, or the minimal order, FAP was first introduced in the 20th century when channels were sold as single frequencies [4]. The objective was to find the assignment that meets all interference constraints and all requests using the smallest number of frequencies. This is formulated mathematically using a binary variable x_{ij} that defines whether transmitter e_i is using frequency f_j or not.

$$x_{ij} = \begin{cases} 1, & \text{if the frequency } f_j \text{ is assigned to the transmitter } e_i, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

The objective is to minimize the number of frequencies such that all requests are fulfilled for each transmitter:

$$min \sum_{j=1}^{Card(F)} \sum_{i=1}^{n} x_{ij}, \quad (8)$$

such that

$$\forall i, 1 \le i \le n, \ \sum_{j=1}^{Card(F)} x_{ij} = R_i. \quad (9)$$

Equation (8) can be transformed by introducing a variable y_j that indicates the use of frequency f_j:

$$\forall j, 1 \le j \le Card(F), y_j = \sum_{i=1}^{n} x_{ij}. \quad (10)$$

The new objective formulation is as follows:

$$\min \sum_{j=1}^{Card(F)} y_j. \quad (11)$$

(ii) Spectra of used frequencies

When frequencies began to be purchased as bandwidth, network providers needed to know the range of frequencies that they could use. This created the minimal span FAP, formulated as follows [30]:

$$min\ (f_{max} - f_{min}), \quad (12)$$

such that the requests are fulfilled according to the conditions

$$\forall i, 1 \le i \le n, \ \sum_{j=1}^{Card(F)} x_{ij} = R_i. \quad (13)$$

And

$$f_{min} = \min_{1 \le i \le n, 1 \le j \le Card(F)} (f_j * x_{ij}), \quad (14)$$

$$f_{max} = \max_{1 \le i \le n, 1 \le j \le Card(F)} (f_j * x_{ij}). \quad (15)$$

An alternative formulation is applied if the smallest frequency used is designated as ‘1’, thus allowing the reduction of (12) to the maximum-used FAP [31]:

$$min\ f_{max}. \quad (16)$$

Other formulations are known [32–35]. The minimal order and minimal span problems may not have interference-free solutions. If this is the case, one of the objectives described in the remainder of this section may be optimized.

(iii) The number of violations

One option is to find an assignment in which the number of interferences (constraint violations) is minimal. This is called the minimal interference or minimal violation FAP [30]. The following objective function is used when a transmitter pair e_1, e_2 is assigned frequencies f_1, f_2:

$$\min\Big(\sum_{f_1=1}^{|F|} \sum_{f_2=1}^{|F|} \sum_{e_1=1}^{n} \sum_{e_2=e_1+1}^{n} x_{f_1 e_1} * x_{f_2 e_2} * \mathrm{Inter}(f_1, e_1, f_2, e_2)\Big), \quad (17)$$

where

$$\mathrm{Inter}(f_1, e_1, f_2, e_2) = \begin{cases} 1, & \text{if } |f_1 - f_2| \le I_{e_1 e_2}, \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

Each request is assigned a frequency as in (13).

(iv) Cumulative interferences

Minimizing the number of interferences may be an unhelpful criterion if interferences are minor and tolerable. In this situation, the amount of interference measured as the signal-to-interference ratio may be more useful. The objective is to minimize the sum of the interferences. This is called the minimal cumulative (or sum of) interferences problem [12].

Using the interferences matrix M (described in Section III.B.), the objective is formulated similarly to (13) and (18) as follows:

$$min\Big(\sum_{f_1=1}^{Card(F)} \sum_{f_2=1}^{Card(F)} \sum_{e_1=1}^{n} \sum_{e_2=e_1+1}^{n} x_{f_1 e_1} * x_{f_2 e_2} * Inter(f_1, e_1, f_2, e_2) * M_{e_1 e_2}\Big). \quad (19)$$

However, if M is defined as in previous studies [16, 17] (see Section III.B.), the sum in (19) must be maximized (or its negative value minimized), since the interference measurement decreases when the mean value of the matrix $M(\mu_{ij})$ increases. Extensions of this approach have been proposed that consider a quality threshold, a value that represents an acceptable signal-to-noise ratio [36–38].

The minimal cumulative interference problem may be considered as a generalization of the minimal interference problem [39, 40]. For frequencies f_i and f_j assigned to transmitters e_i and e_j, one measurement of interference would be the difference between the actual separation between these frequencies and the minimal allowable distance between constraint violations. The objective function is as follows:

$$min\Big(\sum_{f_1=1}^{Card(F)} \sum_{f_2=1}^{Card(F)} \sum_{e_1=1}^{n} \sum_{e_2=e_1+1}^{n} x_{f_1 e_1} * x_{f_2 e_2} * max\big(|f_1 - f_2|, I_{ij}\big)\Big). \quad (20)$$

Another way to consider cumulative interferences is to attribute a weight or penalty to each constraint [41, 42]. The objective is then to minimize the sum of the weights associated with the violated constraints. This may result in a larger number of less critical violations. Weight factors can be defined at more than one level, depending on whether a constraint is violated by one channel or multiple channels.

Minimizing the weighted sum $\alpha|V| + \beta|W|$ of the co-site ($|V|$) and the remaining constraint violations ($|W|$) has been considered [15]. Usually, $\alpha > \beta$ since serious interferences are caused more often by channel selection constraint (CSC) violations than by other constraint violations [12]. Another formulation with multiple levels has been described elsewhere [43].

(v) Cost of cumulative interferences

In the minimal interferences cost FAP, the objective to be minimized is the sum of the costs that will be paid for every violated interference constraint [4]. For each pair of transmitters e_1 and e_2 assigned frequencies f_1 and f_2, the cost to be paid is defined as follows:

$$min\left(\sum_{f_1=1}^{Card(F)} \sum_{f_2=1}^{Card(F)} \sum_{e_1=1}^{n} \sum_{e_2=e_1+1}^{n} x_{f_1 e_1} * x_{f_2 e_2} * Inter(f_1, e_1, f_2, e_2) * C_{sig}(e_1, e_2)\right), \quad (21)$$

such that all the requests are fulfilled as in (13), where C_{sig} is the cost function (see Section III.B.). In this problem category, only the costs that exceed a given threshold are considered. Note that in the minimal interference, minimal cumulative interference, and minimal interference cost approaches, all communication requests are fulfilled completely [17–19].

(vi) Number of blocked calls

In some markets, service quality is critical and interferences cannot be tolerated. The constraint becomes the number of communication requests fulfilled. This forms the minimal blocking or maximal service FAP [44]. If for a given request, no frequency can be assigned without creating interferences, this request will be rejected. The objective of minimizing the number of rejected or blocked requests is formulated as follows:

$$min\left(\sum_{i=1}^{n}\left(R_i - \sum_{j=1}^{Card(F)} x_{ij}\right)\right). \quad (22)$$

In this case, the solution is interference-free.

(vii) Network perturbation

Once an initial frequency assignment plan is established, channel allocation and release must be updated continually, as new requests arrive or ongoing communications end [9]. The objective then becomes to reduce the number of changes made to the initial assignments when managing variations in demand. This creates the minimal perturbation FAP [45, 46].

The different objectives of a FAP can be optimized individually in single-objective optimization or jointly in multi-objective optimization. Single-objective optimization has been examined previously in minimal order [47], minimal span [7, 8, 32–35], minimal interference, [3, 11, 27] and other FAPs [18, 19, 36–38]. In this form, the result is the optimal or nearly optimal frequency plan. Table 1 summarizes the types of FAPs with an indication of the existence of a solution and whether it has been discussed yet by the scientific community.

Multi-objective optimization typically provides superior solutions. In most applications, ensuring high-quality service at the lowest cost requires optimizing more than one objective. Multi-objective optimization has started to receive more attention in the frequency assignment literature but remains limited to considering two objectives, for example, minimizing the number of constraint violations and the corresponding sum of interferences [48], or combining the minimal order and minimal span problems. The output in the latter case is a solution or set of solutions depending on the decision-maker's choice and the form of optimization (a priori, a posteriori, or interactive) [20].

Table. I

FEASIBILITY OF SOLUTIONS TO THE DIFFERENT CATEGORIES OF FREQUENCY ASSIGNMENT PROBLEMS (FAPS).

Assignment Problem	Existence of Solution (Yes/No)	Under Discussion (Yes/No)
Minimal order	No	Yes
Minimal span	No	Yes
Minimal interferences	Yes	No
Minimal sum of interferences	Yes	No
Minimal interferences cost	Yes	No
Minimal blocking	Yes	No

The strength of multi-objective optimization has motivated researchers to develop a multi-objectivized version of the FAP [49]. In this form, the problem is treated as a single-objective minimization task in a multi-objective framework with a main objective and additional objectives that measure diversity in terms of the distance to the closest neighbour [50] or the average distance to all individuals, and which may be maximized [51–53].

V. MODELS OF THE FREQUENCY ASSIGNMENT PROBLEM

The task of modelling real frequency assignment problems is complex due to the variation between the policies and preferences of telecommunications providers. Graph theory and constraint satisfaction are the main approaches employed in the literature.

A. Graph theory models

In graph theory models for FAPs, a simple undirected graph $G = (V, E)$ is considered, defined by a set of nodes V (also called vertices) and a set of edges E that connect the nodes. The graph G can also incorporate other information describing the peculiarities of the model.

The most widely used model of this type is graph colouring, which consists of assigning an integer colour to each node so that no two adjacent nodes (connected directly by an edge) receive the same colour and so that the number of colours used is minimized, this number being the 'order' of the solution (colouring).

The problem can be formulated as finding a function $C : V \to \mathbb{N}, i \to C_i$, such that:

$$\forall (i,j) \in E, |C_i - C_j| > 0. \quad (23)$$

Figure 4 illustrates an example of a simple graph and a feasible colouring with an order of three.

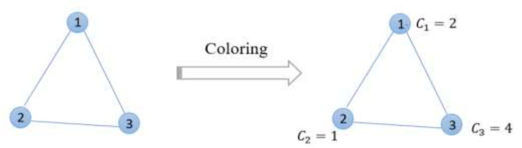

Fig. 4: A simple graph and a feasible colouring of its vertices..

Graph colouring for the FAP was first introduced in the 1970s [54, 55] and has since received much attention [56–59]. Each node represents a transmitter, and each edge connecting two nodes represents either an interference risk or requests between the transmitters, and hence a separation requirement to be satisfied between the pairs of colours (frequencies) assigned to the nodes. The minimal order problem described in III.B. can be modelled as a graph colouring problem when the following conditions hold:

- Each transmitter has exactly one request:

$$\forall i, 1 \leq i \leq n, R_i = 1. \quad (24)$$

- All transmitter nominal sets F_i are the same and equal to the global set F, which is the set of integers:

$$\forall i, 1 \leq i \leq n, F_1 = F_2 = \cdots = F_n = F = \mathbb{N}. \quad (25)$$

- All interference separations are equal to 0. This can be expressed as follows:

$$\forall\ interfering\ i \text{ and } j \in [1, n], |f_i - f_j| > 0. \quad (26)$$

Since simple graph colouring does not allow a complete representation of the FAP, several generalizations have been proposed [4], each seeking to meet additional FAP modelling requirements.

(i) Set or list colouring problem

One generalization assumes that, given a simple graph $G = (V, E)$, each node i of V is associated with a list L_i of allowed colours [51, 52]. The lists are not necessarily of the same length. The list colouring problem is then to assign to each vertex i of V a colour C_i from its set L_i so that two adjacent vertices receive different colours and the number of colours used is minimized. Mathematically, the set colouring problem is to find a function $C : V \to \mathbb{N}^+, i \to C_i$ such that relationship (23) is satisfied. The standard graph colouring problem is the special case in which all the sets Li are the same. Set colouring is a perfect model for the minimal order problem when the conditions (20) and (22) apply and the sets F_i differ from one transmitter to another (condition (25) is not met).

Such as $\mathbb{N}^+$ is the set of natural numbers.

(ii) Multi-colouring problem

This generalization incorporates into a simple graph $G = (V, E)$ a set of demands $D = \{d_i, 1 \leq i \leq n\}$ that defines, for each node i, the required number d_i of distinct colours. The task is to assign the colours so that adjacent nodes do not receive similar colours. The order must be as small as possible. The problem can be formulated as finding a function $C : V \to \mathbb{N}^+$, $i \to C_i$ such that

$$\forall i, 1 \leq i \leq n, C_i = \{C_{i1}, C_{i2}, \ldots, C_{id_i}\} \quad (27)$$

and

$$\forall (i, j) \in E, \forall 1 \leq k_i \leq d_i, \forall 1 \leq k_j \leq d_j:$$

$$|C_{ik_i} - C_{jk_j}| > 0. \quad (28)$$

This models the minimal order problem when conditions (25) and (26) apply, as described in [60, 61].

(iii) T-colouring problem

In T-colouring, the simple graph $G = (V, E)$ is weighted [4, 7]. A set D_{ij} of non-negative integers containing 0 is assigned to each edge E so that the absolute difference between the colours assigned to vertices i and j does not belong to D_{ij}. The problem is equivalent to finding a function $C : V \to \mathbb{N},\ i \to C_i$ such that

$$\forall (i, j) \in E, |C_i - C_j| \notin D_{ij}. \quad (29)$$

If the sets D contain consecutive integers, condition (29) reduces to the following and is called a restricted T-colouring problem [62, 63] or bandwidth colouring problem [64]:

$$\forall (i, j) \in E, D_{ij} = \{0, 1, \ldots, d_{ij}\}, |C_i - C_j| > d_{ij}. \quad (30)$$

The task is to find a feasible colouring of the vertices of G using the smallest colour bandwidth, called the 'span' [65–68]. This can model the minimal span problem when conditions (24) and (25) are met.

Many combinations of the above generalizations have been studied: set T-colouring, list T-colouring, multi-set-colouring [69–71] and various transformations [9, 56].

B. Constraint satisfaction problem models

The constraint satisfaction framework is now used extensively to describe and solve many optimization problems in which some constraints must be satisfied while the others must be prioritized, as is the case in planning, scheduling, and design. It has been applied to model [72], then to improve [73] the lower bound on the overall valuation of any partial frequency assignment. Table 2 summarizes the types of graph colouring that have been applied to FAPs along with a description of the separation constraints set, the number of nodes that are required, and the corresponding nominal set for each type of problem.

Table. II
SUMMARY OF THE FEATURES OF FAPS SUPPORTED BY GRAPH COLOURING.

Problem	Separation constraints set	Requested number per node	Nominal sets
Simple graph coloring	{0}	1	$F_1 = F_2 = \cdots = F_n = F = \mathbb{N}$
Set coloring	{0}	1	Not necessarily equal
Multi-coloring	{0}	$\mathbb{N}$	$F_1 = F_2 = \cdots = F_n = F = \mathbb{N}$
T-coloring	$\mathbb{N}^*$	1	$F_1 = F_2 = \cdots = F_n = F = \mathbb{N}$

Such as $\mathbb{N}^*$ is the set of signed whole numbers excluding 0.

VI. Optimization methods for the frequency assignment problem

As mentioned above, frequency assignment is an NP-hard problem for which an optimal solution cannot generally be found in polynomial time. Furthermore, the current large number of applications in the telecommunications field and the limited frequency spectrum make interference-free assignment difficult and often impossible. Consequently, most recent studies propose approximately optimal solutions or a compromise between the different objectives. Upper and lower bounds of the objective function and some exact methods have nonetheless been proposed under special conditions. In this section, we present and categorize the proposed approaches to solving variants of the FAP.

A. Exact methods

Table. III
Feasibility of solutions to the different categories of frequency assignment problems (FAPs).

References	Methods	Objectives
[29]	Tree search algorithm	Interference handling
[46]	Integer linear programming with exact branch-and-cut technique based on strong cutting planes	Achieve an acceptable signal-to-noise ratio
[56]	Exact enumerative method	Achieve an acceptable signal-to-noise ratio
[74]	Quadratic 0–1 integer programming	Reduce interference
[75]	Two-layer conflict graph and branch-and-bound method	Partial constraint satisfaction
[76]	Russian doll search method	Bandwidth reduction
[77]	Polyhedral structure and {0, 1} linear programming	Maximum satisfiability
[78]	Strong-edge graph coloring and butterfly, Benes, hyper tree, and honeycomb	Minimal required number of frequencies
[79]	Scheduling algorithm	Reduce the interference
[80]	Game theory	Reduce channel interference
[81]	Binary search	Minimizing the frequency utilization
[7], [64], [82], [85]	Graph theory, labelling, and colouring	Minimize the interference and the spectrum of used channels and maximize the throughput
[86]	Coherent Ising machine	Channel distribution with total-throughput maximization

Exact methods have been extensively used in the literature to solve various optimization problems and more specifically the FAP problem. Table 3 summarizes the exact optimization methods along with the optimization objectives that have been examined in the literature.

One of the most extensively studied methods of frequency assignment in the literature is the tree search algorithm [29]. This technique has two main steps: first constructing a tree and choosing a search process such as depth-first or best-first, then processing the tree nodes using reduction techniques such as cutting-plane algorithms or combinatorial lower bounds. The tree search algorithm is applicable to all FAP variants, since they all share an important structure, namely frequency assignment and interference handling. However, their objective functions differ, and lower and upper bounds must be determined using appropriate means for each model.

Quadratic 0–1 integer programming has been used to solve interference cost minimization with some success compared to published benchmarks [74]. An exact tree search algorithm called BBFAB includes all elements mentioned above in a method that introduces a family of lower bounds and three reduction/dominance rules to strengthen and reduce the size of the problem.

A mathematical formulation of the FAP for mobile radio systems in a geographical region has been presented as an integer linear program using an exact branch-and-cut technique based on strong cutting planes [46]. Tested on 85 real-world instances provided by the Centro Studi E Laboratori Telecomunicazioni (CSELT), this technique was able to provide the exact solution of instances with up to 203 nodes in acceptable computation time.

The authors of [56] introduced an algorithm for frequency assignment in mobile phone, radio, and television systems. Frequency allocation must meet interference requirements to achieve an acceptable signal-to-noise ratio. To this end, an exact enumerative method was used by setting criteria that reduce the size of the instances. The well-chosen sub problem algorithm was used to solve larger instances by extending the obtained solutions. Applied to real-life situations, this approach has produced better results than previously reported methods.

The assignment problem has been modelled as a graph colouring problem in the cognitive radio network context, and solved using efficient centralized and distributed sequential algorithms based on dynamic vertex ranking with novel cognitive-radio-specific saturation metrics [7]. Interference susceptibility is represented as a two-layer conflict graph that determines the interference potential using the weights of the co-channel and adjacent channel edges. A dynamically updated local list of locked channels in the interference zone protects the primary users. In addition, frequency and bandwidth selection are considered instead of channel selection in the frequency assignment decision process.

The authors of [75] solved the partial constraint fulfilment problem using a branch-and-bound method, which was described using a genetic algorithm. In [76], the authors present the Russian doll Search method, which substitutes one search with n subsequent searches on nested subproblems. This resulted in good performance in both small and large cases.

In [77], the polyhedral structure of the partial constraint satisfying problem is investigated as a potential model of FAPs. The authors express the partial constraint fulfilment problem as a {0, 1} linear programming problem and prove theorems for lifting aspects of a sub problem to facets of the entire problem.

The minimal required number of frequencies has been determined for interconnection networks, namely the butterfly, Benes, hyper tree and honeycomb networks, by treating the FAP as a strong-edge graph colouring problem [78].

To reduce interference in broadband wireless networks, an efficient channel assignment algorithm has been proposed to avoid interference in multi-channel wireless mesh networks by optimizing the number of channels [79]. The algorithm uses scheduling to avoid primary interference, reduces the transmission delay for uplink/downlink networks, and prioritizes to prevent packet accumulation in multi-hop networks.

A game theoretic framework has been used to obtain a frequency assignment algorithm for networked unmanned aerial vehicles, whose optimality and computational complexity were validated through numerical simulation [80]. To maintain access to military communications and space navigation for combatant commanders, frequency assignment in two-way traffic across a link using two disjoint bands has been studied [81]. Two variants of this problem were considered as separate tasks: (i) minimizing the frequency utilization of the more congested air platform in the two frequency bands while meeting the traffic demand, without violating the frequency constraints, and (ii) the minimal span problem of minimizing frequency utilization in both bands. An optimal minimal span solution was found using a binary search, providing a feasible solution to the first task, which is a sub problem of the other, thus reducing the computational complexity of the solution.

A model that fits the FAP into graph theory is $L(3,2,1)$-labelling. This model assigns integer frequencies to radio transceivers such that (i) transceivers located one unit apart get frequencies that differ by at least three, (ii) transceivers located two units apart get frequencies that differ by at least two, and (iii) transceivers located three units apart get frequencies that differ by at least one. A generalization of (s,t)-relaxed k-$L(2,1)$-labelling is used to establish the optimal (s,t,r)-relaxed k-$L(3,2,1)$-labelling for paths and certain cycles and for hexagonal and square lattices, thus constituting a solution for the FAP in these three cases [82, 83].

A method of finding the optimal solution to the channel assignment problem of mobile communication systems, with multiple base stations requiring the same number of channels, uses n-fold t-separated $L(j_1, j_2, \dots, j_m)$-labelling when the corresponding interference graph consists of triangular and square lattices, and consecutive labelling when the corresponding interference graph is a complete graph or cycle [84].

The FAP has been also formulated as k-$L(p,1)$-labelling of graphs [61, 64], which involves finding a function $f: V(G) \rightarrow \{0,1,2,\dots k\}$ such that $|f(u) - f(v)| \geq p$ if $d(u,v) = 1$ and $|f(u) - f(v)| \geq 1$ if $d(u,v) = 2$, where $d(u,v)$ denotes the distance between vertices u and v. In the case of planar graphs without 4 or 6 cycles, the authors of [85] propose upper bounds that are obtained in a constructive manner that can be harnessed as a labelling method for the channel assignment problem and for other types of labelling and colouring problems.

Ultra-fast and rigorous channel distribution has been achieved via an entirely different approach using a coherent Ising machine for dense WLAN systems [86]. Based on a high-speed laser system, this material technique uses optoelectronic properties to solve Ising problems. The FAP is formulated first as a total throughput maximization problem for large-scale centralized WLAN systems, and then converted to an Ising machine problem. Experimental results obtained within milliseconds (‘ultra-short’ times) confirmed that total throughput was thus improved.

B. Greedy algorithm-based methods

A greedy algorithm is a constructive process whereby a solution is built piece by piece, selecting the best option at each stage. For FAPs, a greedy algorithm assigns frequencies iteratively by selecting a vertex and then assigning to it the best feasible frequency that optimizes the global objective function. The resulting solution is usually adequate but only sometimes optimal.

Distributed greedy and game theoretic approaches to the channel allocation and sharing problem have been considered in cognitive radio mesh networks for Internet of Things systems to maximize network energy efficiency while satisfying scalability and self-organization constraints [87].

A polynomial-time probabilistic greedy algorithm has been proposed for cellular networks using a complete edge-weighted FAP graph in which every call is represented by a vertex and an edge weight represents the minimal frequency separation required between calls. The objective is to assign frequencies (colours) to the vertices so that the requested span is minimized. The algorithm performs two main tasks: ordering the nodes in a probabilistic manner and executing an exhaustive vertex colouring strategy using an iterative compression phase to further reduce the obtained span. Its execution time and deviation from optimality compared favourably to the benchmarks [34]. Incorporating randomization and net filter discrimination effects into the communication model has been found to considerably improve the early greedy and fast heuristics [88].

C. Trajectory methods

Among the most common methods of solving NP-hard problems are algorithmic techniques called local search methods, which may be used alone or incorporated into other algorithms to explore solutions sequentially along a rational trajectory through the solution space. This type of approach needs to proceed from an initial feasible solution, which often means beginning by building a heuristic solution.

A real-time solution to the FAP has been sought using mathematical models that incorporate field measurements of real-time interference [18], as practised in existing GSM networks. A set of hybrid metaheuristics created for this purpose, and combining the advantages of global and local search methods, was found to perform better than previous solutions, based on tests with two real GSM networks. Two of the metaheuristics performed better than the others, namely a population-based scatter search algorithm and a trajectory-based (1+1) evolutionary algorithm [85].

A local search heuristic and a compound motion-based heuristic for fixed spectrum frequency assignment in telecommunication networks [89] has been found to outperform the dynamic list-based tabu search [90] and the heuristic manipulation technique [91] in 88% and 79% of tests, respectively.

A revised version of the tabu search algorithm including a dynamic tabu for the fixed spectrum frequency allocation problem is claimed to be efficient in tests against several benchmark sets and existing algorithms [92], improving on some of the best allocations available for COST259 benchmarks and proving very efficient when the best solution is required without constraints on the runtime. This approach is claimed to reduce the number of rejected calls in exchange for a small increase in interference.

D. Population-based methods

Metaheuristics are techniques for searching a space of feasible solutions in a structured manner to determine near-optimal solutions in a short time. They provide no guarantee of optimality, are often derived from natural processes, and are increasingly hybridized with other methods to mitigate their weaknesses. In particular, a strategy for avoiding getting stuck in a region of the search space may be necessary.

To solve the FAP, two iterative strategies of implementing an ant colony system for the graph colouring problem have been proposed, namely construction of feasible solutions and solution improvement through tabu search [57]. In extensive comparisons, this approach outperformed hybrid graph colouring algorithms on large instances of the Dimacs benchmarks.

The T-colouring problem is a generalization of the classical graph colouring problem. It has been applied to frequency assignment using an algorithm inspired by the mating process in real bee colonies [60] in order to solve the graph colouring, restricted T-colouring, and general T-colouring problems [7, 8]. A hybrid evolutionary approach combining the ant colony algorithm and tabu search is also described. A comparison between bee mating-based and ant colony-based optimizations demonstrates the effectiveness of the method using benchmark methods.

A well-adjusted ant-colony-inspired optimization algorithm has been shown to outperform an evolutionary algorithm at solving the FAP [17]. A continuous-domain ant colony algorithm [93] inspired by the natural behaviour of Pachycondyla apicalis ants [94] has also been found to be highly suitable in computations of standard problem instances, in direct comparison with tabu search and simulated annealing.

An extensive study of three hybridizations of the stochastic competitive Hopfield neural network and the genetic algorithm showed that when the neural network manages the constraints and the genetic algorithm searches for the highest-quality solutions with the lowest costs, the performance is similar or superior to other algorithms on five benchmark problems and 12 large, randomly generated problems [95].

Parallel cooperation between ‘marriage in honey bees’ optimization and two ant colony strategies for solving the graph colouring problem has been studied [96]. Various collaboration and parallelization modalities, executed using a simulated parallel machine on a cluster of PCs, showed that one ant colony-based construction strategy was faster than the other methods, thanks to the parallel implementation. Combining the two strategies gave better results than either algorithm alone.

The ability of artificial immune systems to solve optimization-related problems has been harnessed to design a cellular frequency allocation model [27] that was found to be efficient and able to reuse available non-conflicting frequencies. Another approach is combining particle swarm optimization with tabu search, which is claimed to reduce premature convergence to a local minimum to achieve global optimization [13].

A new low-cost model has been implemented to reduce optimization solution storage volume and thus evaluation time. In tests against most benchmarks, the proposed algorithms were found to be superior to particle swarm optimization, tabu search, ant colony optimization, simulated annealing, and genetic algorithms [22, 97–99] in terms of convergence rate, number of iterations, solution storage volume, and execution time needed to converge to the optimal solution.

An algorithm that ensures frequency allocation in cells of a cellular network has been obtained by hybridizing the Hopfield neural network and the tabu search algorithm, while adopting the principles of short-term memory and the tabu search candidate list in the control of the neural selection process [100].

A diversity management strategy based on replacement has been applied to overcome the premature convergence that occurs when using some of the most recently developed evolutionary algorithms [101]. In addition to the possibility of transforming a single-objective problem into a multi-objective problem by considering diversity as an additional objective, the proposed extensions were tested on 44 instances of two FAP variants, which led to the updating of the benchmark method, based on superior solutions (slowing of population convergence) for 11 instances.

Three graph theoretic heuristics, namely greedy, greedy colouring-based, and clique colouring-based, have been tested for the joint assignment of antenna frequency and polarization in a cluster-based tactical communication environment [23]. These heuristics are of low complexity and are quasi-optimal in terms of chromatics and thus very suitable for practical use. When executed in artificial degenerating environments, the coloring-based approach clearly outperformed the greedy algorithm, with few exceptions. A clique coloring-based algorithm was able to find the optimal solution.

A particle swarm optimization augmented by an approach called guided search, designed to minimize an interference cost, was found to improve the particle location updating operation and thereby avoid local minima [40]. This enhanced the convergence rate, robustness, and stability of the results significantly. A gravitational search algorithm [102, 103] tested against benchmark combinatorial algorithms for military applications was also found to be satisfactory [104].

The static version of the single-objective routing and spectrum assignment problem in flexible-grid optical networks has been tackled using an efficient genetic algorithm-based metaheuristic described as providing neo-optimal solutions [105]. Compared to exact methods based on integer linear programming and other classical heuristics, the proposed method generated optimal solutions in a short execution time in a variety of scenarios [106].

An initial attempt to use an adapted version of the harmony search algorithm to solve the minimal interference problem was unsuccessful due to premature convergence and getting stuck repeatedly in local optima [73]. This was overcome by combining the algorithm with local search and opposition-based learning. Although the resulting performance was comparable to the literature results, the method needs improvement to be competitive.

A hybrid heuristic with no problem-specific parameters (unlike most metaheuristics) has been proposed, based on the non-dominated sort-II genetic algorithm and tabu search [107]. At each iteration, tabu search is applied to each element of the population to be substituted by the best solution in its neighbourhood, and the elements of the next generation are then selected using the non-dominated sorting algorithm considering the interference and entropy of each solution. Experimental tests indicate that the proposed heuristic provides higher-quality solutions than four existing heuristics and has an equivalent convergence time.

In [108], the channel assignment problem was formulated as a time slot allocation problem and solved using a vertex coloring algorithm. This approach offered a significant improvement in network throughput, transmission delay, and packet loss rate compared to distributed coordination function and TMCA algorithms. In a study of the dynamic FAP, new requests were incorporated gradually, and frequencies were allocated to them while minimizing the number of reassignments [108]. The problem was formulated as a vertex colouring problem in a graph and treated using a heuristic approach based on both the canonical correlation analysis algorithm and a process for slot reservation. On dynamic datasets generated from static benchmark datasets, performance was superior to the state-of-the-art approach.

The problem of frequency distribution between wireless radio stations operating in a high- and a low-frequency band has been addressed using a model of the electromagnetic compatibility problem [109]. An effective solution was thus obtained faster than by the conventional backtracking method.

Game theory has been combined with centralized and distributed behaviour, while respecting net filter discrimination, to obtain a heuristic for allocating and utilizing frequency resources in communication systems [110]. This allowed maximizing the sum of the user signal-to-interference-to-noise ratios, minimizing the standard deviation thereof, and reducing the computational complexity.

The minimal interferences problem has been solved using three metaheuristics: variable neighbourhood search, stochastic local search, and a hybridization of these two, with the latter incorporated as a subroutine [111]. In empirical experiments, the hybrid method gave higher-quality solutions than either search method alone and offered an acceptable trade-off between scalability and diversification.

Particle swarm optimization of static channel assignment in maritime communication networks has been shown to be efficient at reducing interference in multi-radio simulations performed in NS-3, considerably improving the performance of both sparse and dense networks with fewer iterations than in previous studies [112]. The channel assignment was found to modify the routes, which can lead to unnecessary channel changes. A newer swarm-intelligence metaheuristic [113], based on an algorithm inspired by fireworks explosions in the night sky [114], has been shown to converge quickly to global optimal solutions to the FAP.

The dynamic FAP has been modelled as a combinatorial optimization problem, by separating it into two phases to develop a metaheuristic method of finding the solution for a tactical wireless communication network [115]. This involved optimizing the neighbourhood search process and using simulated annealing and greedy algorithms to facilitate the frequency regularization process and generate new solutions with fewer frequencies and reduced influence of interferences. The superiority of a simulated annealing approach to solving the minimal interference problem has been demonstrated in several instances [116] using two benchmarks [44, 117]. This study provided the first known instance/solution database for this FAP.

The dynamic FAP is studied in [118], in which new requests are gradually introduced and need frequency assignments. A heuristic approach is proposed to solve this problem using different solution methods. The gap technique is also used to improve efficiency.

The authors of [119] describe the FAP by considering the electromagnetic compatibility (EMC) problem. They concentrate on the many types of interference. When utilized in military situations, the authors discovered that their method was faster than the standard counterpart (the backtracking method).

A modified version of the recently described immune plasma algorithm, a metaheuristic inspired by a technique used to process SARS-CoV-2 coronavirus convalescent or immune plasma [120], has been proposed as a more efficient solution to the channel assignment problem [121] compared to brute force search. The latter work proposes a fairness-oriented semi-chaotic genetic algorithm-based channel assignment technique to solve the node starvation problem in wireless mesh networks. Unlike previously published methods, this technique, applied with a nonlinear fairness-oriented fitness function, ensured maximal link fairness and minimal link interference. A multi-criteria link-ranking channel assignment algorithm was used to create the primary chromosome engendering a population that searched for global minima. High node equity was maintained while network resource use efficiency was increased by up to 23%.

E. *Multi-objective methods*

Methods dedicated to solving the multi-objective frequency allocation problem are summarized in this subsection.

Approximation algorithms, namely tabu search, simulated annealing, and DDS, have been applied to simultaneously reduce the total interference cost and the frequency spectrum usage in the absence of interference [122], and have been compared in practical instances to using a nonlinear relaxation lower-bound technique to evaluate solution quality [123].

In [17], a multi-objective memetic algorithm based on the non-dominated genetic sorting algorithm II was applied to a single-objective FAP with an artificial objective attached to circumvent stagnation. This approach was validated in tests on real-world instances and shown to provide high-quality solutions in reasonable time and to improve on the best previously known frequency plans in those instances.

In [124], a variant of the multi-swarm particle swarm optimization (MS-PSO) method was proposed to optimize a bi-objective FAP [49]. A condition to be satisfied by the optimal solution was described and incorporated into the conventional MS-PSO algorithm to avoid convergence to a local minimum and to define the search direction. Simulations generated solutions that provide a best-fit compromise between the two objectives, and the embedded condition yields better solutions compared to the conventional MS-PSO algorithm [125, 126].

In another line of research, [127] focuses on the aspects and characteristics of real GSM networks to propose a hybrid parallel model that combines an island-based model with a hyper-heuristic to address a multi-objective version of the FAP. A thorough robustness analysis of the proposed model, as well as the results of its application to several real network instances, demonstrate its efficiency in obtaining high-quality solutions in a short time. Moreover, it offers a remarkable improvement over the best-known frequency plans in the literature.

An ant colony optimization method has been described for multi-objective channel assignment aimed at achieving high spectral efficiency in mobile ad hoc networks with a clustered topology, through a trade-off between maximizing spectral utilization and minimizing interference [128]. Stable and scalable solutions were thus found with a minimal number of assigned channels, independently of the number of network clusters.

The numerical parameters of a diversity-based multi-objective evolutionary algorithm have been fitted specifically to the FAP using a hybrid control scheme based on both fuzzy logic controllers and hyper-heuristics [28]. Based on evaluation in benchmark instances and real-world cases, this approach produced equivalent or superior frequency plans to those delivered by a significant number of standard diversity-based multi-objective configurations.

The problem of channel assignment in multi-cell wireless local area networks has been formulated as a mixed-integer linear program aiming to minimize the worst-case interference at the network access points [129]. Simulations showed that a proposed optimization model achieves the objective with better fairness than the pick-first greedy algorithm, the optimal model, and the max-min model.

The preeminent notions of the Nash equilibrium from game theory and Pareto optimality have been combined to obtain a hybrid multi-objective genetic algorithm for solving the multi-objective FAP in mobile networks [130]. Several tests performed on benchmark instances (Philadelphia FAP and COST259) showed that this method provides high-quality solutions that are both Nash equilibria and Pareto non-dominated. Its performance was confirmed in comparisons with the most-referenced multi-objective optimization algorithms, such as the non-dominated sorting genetic algorithm II (NSGA-II) and the improved version of the strength Pareto evolutionary algorithm (SPEA2) on the same instances. A multi-objective immune memetic algorithm has been described for solving the FAP in cellular networks with the goal of minimizing network total interference, peak interference, and the number of frequencies used [131]. The proposed approach replaces the crossover and mutation evolutionary processes with a local FAP-specific search strengthened by a guided diversification strategy. The algorithm's exploration and exploitation capabilities are further enhanced by clonal selection inspired by artificial immune systems.

A hybridization of a multi-objective memetic algorithm and an artificial immune system hybrid algorithm has been investigated for solving the multi-objective FAP in cellular mobile networks [132]. The objectives considered simultaneously are minimizing total interference, peak interference, and the number of frequencies used. Exploration and exploitation capabilities were improved by integrating a local FAP-specific search into the evolutionary process, clonal selection, and receiver editing. Tests on COST259 instances showed the effectiveness of the proposed approach in generating high-quality solutions complying with the hyper volume metric. These results have found support in comparison with the most frequently cited algorithms in the literature. The impact of the key parameters and their relationships were analysed using statistical tools.

VII. Conclusion

We have surveyed the available literature on the FAP, a critical technical challenge for several industries, with specific focus on network applications. The definitions of the problem and its variants as well as its myriad proposed solutions are summarized and presented along with their mathematical formulations. The FAP has been modelled using a wide variety of approaches. The literature is replete with descriptions of optimization-based methods used to find exact and approximate solutions to the FAP. This review is expected to be a reference and a guide for researchers and practitioners deal-ing with or interested in channel frequency allocation and management problems. It allows for the identification of prospective improvements in mathematical formulations and solutions, as well as potential advancements and contributions to the field.

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