Kinematic Modelling of Humanoid Robot Based on Vectorial Approach

Nacer Hadidi, Chawki Mahfoudi, Mohamed Bouaziz, Zaharuddin Mohamed, and Ahcene Bouzida

Abstract− The present work is dedicated to the study of the inverse kinematics solution of a humanoid bipedal robot with 30 degrees of freedom using a newly approach based on vectors that generated from the geometric space of the robot. The solutions of the inverse kinematics problem for all joints articulations are carried out through a simple and linear equations. Finally, simulations of results are performed through harmonization of trajectories of end effectors of the robot's members.

***Keywords*— biped robot, Humanoid robot, inverse kinematics, robotic, vectors calculus.**

NOMENCLATURE

DKM: Direct Kinematic Model.
IKM : Inverse Kinematic Model.
SSP : Simple support phase.
DSP : Double Support Phase.
$C(\theta_i)$: $\cos(\theta_i)$
$S(\theta_i)$: $\sin(\theta_i)$

I. Introduction

Robotics has captivated a portion of the scientific community due to its alluring prospects across various domains [1]. Indeed, robotics has progressed rapidly, transcending what was once considered science fiction into reality. Among the many branches of robotics, humanoid bipedal robots have garnered intense research in recent years [2-4]. Researchers continue to tackle challenges in various aspects, striving for advancements beyond completed studies. Researchers have emphasized various concepts within this domain, including mechanical design, kinematic and dynamic modeling…etc.Mechanical design aims to create human-sized robots with a high degree of freedom within a compact volume, presenting a significant challenge [5], [6]. Consequentelly, mathematical modeling in robotics makes another side of chanllenging, like kinematic modeling. In fact, this last one (kinematic modeling) plays a crucial role in providing the foundational equations used in robot's dynamics and control.

This paper presents an inverse kinematics study of a bipedal robot with a significant number of degrees of freedom (30 DOF). The robot is devided on six mains parts, each parts is modeled in its proper frame. However, this last one is reconducted in the principal frame on the robot's hip which will be expressed on the global frame. Basically, according to the used approach, each angle that represent one degrees of fredom is described by three vectors from which should be expressed by its magnitude and its rotation sense.

II. The Model Of The Biped Robot

The robot model has 30 degrees of freedom (30 DOF) which allow great mobility (Fig. 1).

The robot structure is divided into 06 main parts: the trunk, the neck-head, the upper limbs (right and left arms), and the lower limbs (right and left legs). A primary reference frame is fixed at the hip. Than, all frames within the robot structure are defined according to the Denavit-Hartenberg (DH) convention as it is illustrated in Fig. 1. This allows providing the different matrices that will be used in the Forward Kinematics (Fk). Table 1 summarizes the lengths of the various links of the robot.

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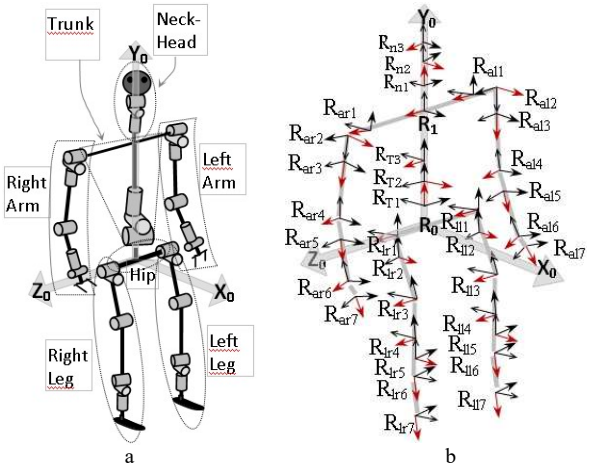


Fig.1: Robot Model. a. Kinematic diagram of the robot. b. Different frames of the robot.

Table 1.
LENGTHS OF ROBOT LINKS

MECHANISMS	ELEMENTS	LENGTHES (m)
Uper limb	Fore-arm	$L_{fa} = 0.28$
	arm	$L_a = 0.30$
	hand	$L_h = 0.10$
Lower limb	Leg	$L_l = 0.30$
	Shin	$L_s = 0.30$
	feet	$L_f = 0.10$
Middle part	Trunc	$L_T = 0.30$
	Hip	$L_H = 0.10$
Head-neck	Head-neck	$L_n = 0.10$

III. KINEMATIC MODELING OF THE ROBOT

Kinematic modeling serves as the foundation in robotics, known through the Forward Kinematic Model (FKM) and Inverse Kinematic Model (IKM).

A. Forward kinematic Model

According to DH convention, the transformation from basic frame to end-effector is expressed by the following matrix given in (1).

$$\begin{bmatrix} s_x & n_x & a_x & P_{dx} \\ s_y & n_y & a_y & P_{dy} \\ s_z & n_z & a_z & P_{dz} \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{0}T_1 \ {}^{1}T_2 \ {}^{2}T_3 \dots {}^{n-1}T_n \qquad (1)$$

$$= [\mathbf{S\ N\ A\ P}]$$

Where:

[S N A P]: exprime orientation and position of end-effector in basic frame. ${}^{i-1}T_i$ is a matrix of homogeneous transformation between two successive frames ($i = 1 \dots n$).

B. Principle of the Inverse Kinematic Calculation Approach.

In this section, we use the new proposed approach that established by [7] which is fonded on a vectorial description of the robot to solve the IK problem. One of the most useful mathematical formula that used to be explored in robotics in provinding analythecal solution of variable articulation β (angle) expressed between two vectors, $\vec{U}$ and $\vec{V}$, is given in (2).

$$\beta = atan2\left(\sin(\vec{U},\vec{V}), \cos(\vec{U},\vec{V})\right) \qquad (2)$$

Subsequentlly, the formula in (2) has an advantage in provinding the correct orientation and value of the angle β at any quadrant of the cercle. In addition, a suitable relation between $\sin(\vec{U},\vec{V})$ and $\det(\overrightarrow{n_U},\overrightarrow{n_V},\overrightarrow{n_W})$ is established by [7].

$$\sin(\vec{U},\vec{V}) = \det(\overrightarrow{n_U},\overrightarrow{n_V},\overrightarrow{n_W}) \qquad (3)$$

Where:

$\overrightarrow{n_U} = \frac{\vec{U}}{U}$, $\overrightarrow{n_V} = \frac{\vec{V}}{V}$ and $\overrightarrow{n_w} = \frac{\vec{w}}{w}$ are, respectivelly, unit vectors of $\vec{U}$, $\vec{V}$ and $\vec{W}$. The vector $\vec{W}$ could be the cross product of $\vec{U}$and $\vec{V}$ or any other orthogonal vector to the plane formed by $\vec{U}$and $\vec{V}$ (UV-plane).

A notable characteristic of determinant regarding its sign emerges when two columns are swapped [8].

$$\det(\overrightarrow{n_U},\overrightarrow{n_V},\overrightarrow{n_W}) = (-1)\det(\overrightarrow{n_V},\overrightarrow{n_U},\overrightarrow{n_W})$$

By knowing, the determinant calculated from the vectors concerning the angle and its cosine value, equation (2) fully determines the angle, including both its sign and value. Hence, equation (2) can be writen as shown in (3)

$$\beta = atan2\left(\det(\overrightarrow{n_U},\overrightarrow{n_V},\overrightarrow{n_W}), \cos(\vec{U},\vec{V})\right) \qquad (3)$$

C. Calculation and identification of angles of the bipedal robot.

As mentioned in the introduction, the biped robot is divided into 06 parts. Therefore, we proceed to model each one separately.

1) Inverse Kinematic Modeling of the Trunk

The movement of the trunk is generated by three joint angles (Fig.2).

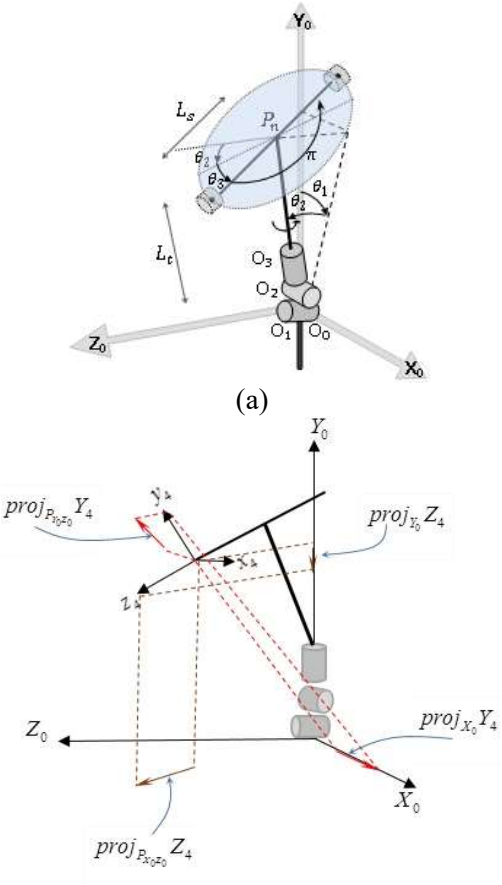


Fig.2: Geometric Description of the Trunk and Shoulder mechanism. a. Geometric Modeling of the Trunk. b. Current Position of the Shoulder.

The joint angle θ_3 is determined by the following three vectors: $proj_{P_{X0Z0}}\overrightarrow{Z_4}$, $\overrightarrow{Z_0}$ et $\overrightarrow{Y_0}$.

Knowing that: $proj_{P_{X0Z0}}\overrightarrow{Z_4} = \overrightarrow{Z_4} - proj_{Y_0}\overrightarrow{Z_4}$. is the projection of the vector $\overrightarrow{Z_4}$ onto the plane $P_{X_0Z_0}$.

$\overrightarrow{Z_4}$ is a direction cosine vector of the desired shoulder position matrix.

So, θ_3 is expressed by (4):

$$\theta_3 = a\tan2\left(det_{\theta_3}, \cos\theta_3\right) \qquad (4)$$

Where: $det_{\theta_3} = det\left[\overrightarrow{Z_0} \ \ proj_{P_{X0Z0}}\overrightarrow{Z_4} \ \ \overrightarrow{Y_0}\right]^T$ and $\cos\theta_3 = \overrightarrow{Z_0}.proj_{P_{X0Z0}}\overrightarrow{Z_4}$

Determining the joint angles θ_1 and θ_2 involves first determining the position of the midpoint (P_n) through the matrix giving the desired shoulder position via the following relationship :

$$\overrightarrow{OP_n} = \overrightarrow{OP_s} - L_s \overrightarrow{A_4}$$

Where
$\overrightarrow{OP_n}$: Vector expressing midpoint of schoulders.
$\overrightarrow{OP_s}$: Vector expressing schoulder position.
L_s : Length of schoulder.
$\overrightarrow{A_4}$: Vector of cosine direction of schoulder's frame on z-axis.

The angle θ_1 is determined by the following vectors: $proj_{P_{X0Y0}}\overrightarrow{n_{Pn}}$, $\overrightarrow{Y_0}$ and $\overrightarrow{Z_0}$.

Given that: $proj_{P_{X0Y0}}\overrightarrow{n_{Pn}} = \overrightarrow{n_{Pn}} - proj_{Z_0}\overrightarrow{n_{Pn}}$ it is the projection of the vector $\overrightarrow{n_{P_n}}$ onto the plane $P_{X_0Y_0}$

Where: $\overrightarrow{n_{P_n}}$ is the unit vector of $\overrightarrow{OP_n}$.

$$\theta_1 = a\tan2\left(det_{\theta_1}, \cos(\theta_1)\right) \quad (5)$$

With : $det_{\theta_1} = det\left[\overrightarrow{Y_0} \;\; proj_{P_{X0Y0}}\overrightarrow{n_{Pn}} \;\; \overrightarrow{Z_0}\right]^T$ and $\cos(\theta_1) = \overrightarrow{Y_0} \,.\, proj_{P_{X0Y0}}\overrightarrow{n_{Pn}}$

Angle θ_2 is determined by the following vectors: $proj_{P_{Y0Z0}}\overrightarrow{n_{Pn}}$, $\overrightarrow{Y_0}$ et $\overrightarrow{X_0}$

Given that: $proj_{P_{Y0Z0}}\overrightarrow{n_{Pn}} = \overrightarrow{n_{Pn}} - (proj_{X_0}\overrightarrow{n_{Pn}})$. It is the projection of the vector $\overrightarrow{n_{P_n}}$ onto the plane $P_{Y_0Z_0}$

$$\theta_2 = a\tan2\left(det_{\theta_2}, \cos(\theta_2)\right) \quad (6)$$

With: $det_{\theta_2} = det\left[\overrightarrow{Y_0} \;\; proj_{P_{Y0Z0}}\overrightarrow{n_{Pn}} \;\; \overrightarrow{X_0}\right]^T$ and

$$\cos(\theta_2) = \overrightarrow{Y_0} \,.\, proj_{P_{Y0Z0}}\overrightarrow{n_{Pn}}$$

2) Inverse Kinematic Modeling of the Neck-Head Mechanism

The movement of the head is governed by the joints incorporated at the robot's neck θ_4, θ_5 and θ_6 as shown in Fig.3.

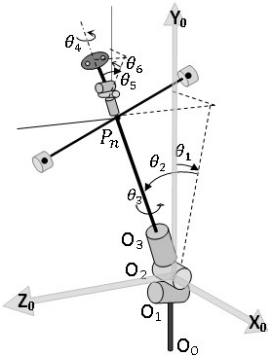


Fig.3: Joints angles of the Neck-Head.mechanism.

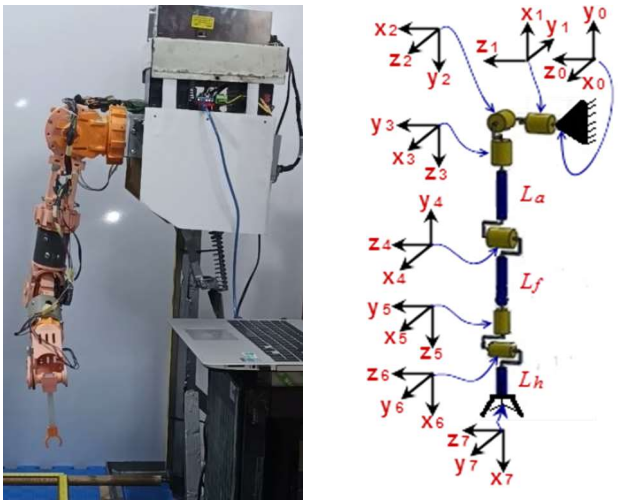
During the robot's movement, the head that is connected to trunk at midpoint by the neck with three joint articulations is assumed to be on the vertical straight position.

Therefore, and from Fig.3, the joints θ_5 and θ_6 must respectively have the same values as θ_2 and θ_1 of the trunk, but in opposite rotation, as expressed in (7). Also, the Joint θ_4 is assumed to be stationary.

$$\theta_5 = -\theta_2 \quad et \quad \theta_6 = -\theta_1 \quad (7)$$

3) Inverse Kinematic Modeling of the Upper Limb (Arm).

Fig. 4 depicts a right upper limb with 6 degrees of freedom of the bipedal robot distributed among three links: the upper arm (L_a), the forearm (L_{fa}), and the hand (L_h)



(a) (b)

Fig.4: Right upper limb of a bipedal robot and assignment of frames according to the DH convention. a. Image of the robot arm. b. inematic chain of the robot arm [7].

In this work, the following assumptions are considered:

- The shoulder, with its 3 degrees of freedom (3DOF), has its rotation axes intersecting at the same point. Therefore, the rotational movement of the shoulder can be treated as a single geometric point.

- Similarly, the same assumption is applied to the wrist, which has 2 DOF.

Fig. 5 depicts a geometric parametrization of the robot (geometric entities) that will be used in the process of solving the IKM.

The strategy to solve the inverse geometric problem is divided into two phases:

- the first phase involves in determining the geometric entities that allowed the determination of the vectors defining each joint.

- The second phase focuses on determining the angles of each joint.

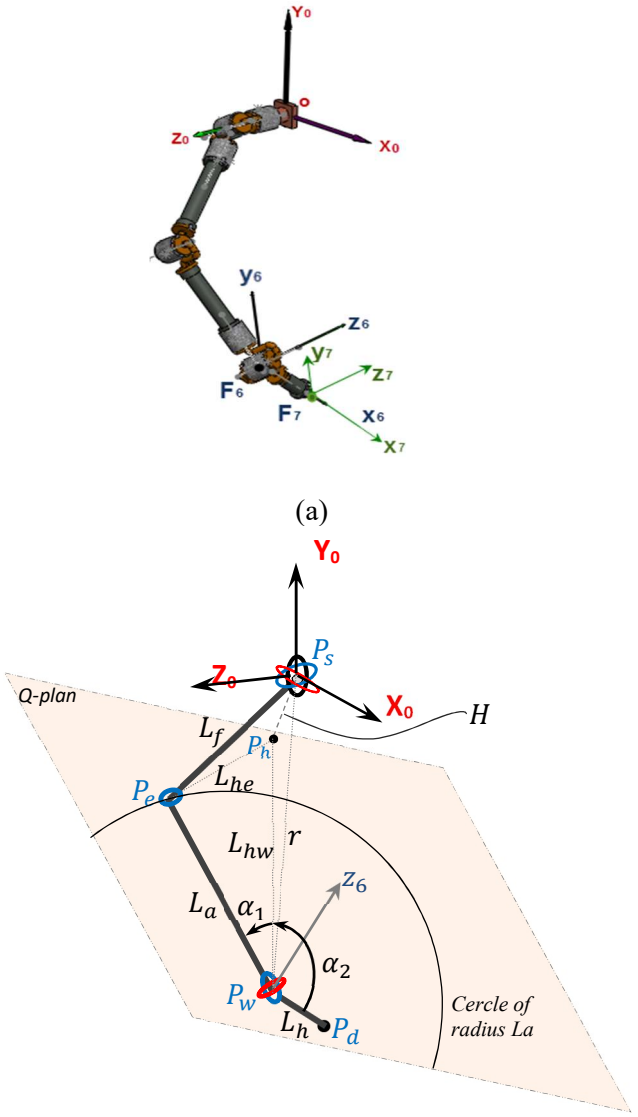


Fig.5: Geometric parametrization of a right upper limb of a bipedal robot. a. Robot configuration with assignment of main frames. b. Geometric entities [7].

a) Calculation of Geometric Entities of the Robot

To do this, the following strategy is adopted:
- Determine the position of the wrist.
- Determine the position of the elbow.

The position of the wrist is given by (8).

$$\overrightarrow{OP_w} = \overrightarrow{OP_d} - L_h\,\overrightarrow{S_6} \qquad (8)$$

where:

$\overrightarrow{OP_w} = (P_{wx}, P_{wy}, P_{wz})^T$ and $\overrightarrow{OP_d} = (P_{dx}, P_{dy}, P_{dz})^T$ respectively are the coordinates of the wrist position and the desired position of the end effector (hand). L_h is the length between wrist and hand. We define the circle of the center P_w (wrist position) and radius L_f (forearm length) on the plane *Q-plan* that is defined by

$$\overrightarrow{OP_c} = \overrightarrow{OP_w} + L_f\cos(\varphi)\overrightarrow{S_6} + L_f\sin(\varphi)\overrightarrow{N_6} \qquad (9)$$

Where:

$\overrightarrow{OP_e} = (P_{ex}, P_{ey}, P_{ez})^T$ and $\overrightarrow{OP_w} = (P_{wx}, P_{wy}, P_{wz})^T$ are, respectively, the coordinates of the circumference of the circle and the center of the circle. $\overrightarrow{S_6}$ and $\overrightarrow{N_6}$ are, respectively, the unit vectors of the x_6-axis et y_6-axis. And φ is the angle between x_6-axis and $\overrightarrow{V_{we}}$ (vector defined by wrist and elbow). Thus, from (9), the position of the elbow is determined by the vector of coordinates given in (10).

$$\overrightarrow{OP_c} = \overrightarrow{V_{oe}} = \left(P_{ex}, P_{ey}, P_{ez}\right)^T \qquad (10)$$

By determining the necessary geometric entities, we proceed for identification of joint angles.

b) *Calculation of Arm Joint Angles*

Geometrically, three vectors (Fig.6), from which two mathematical arguments (arg1, arg2) are expressed and will be used in (6), describe each joint angle.

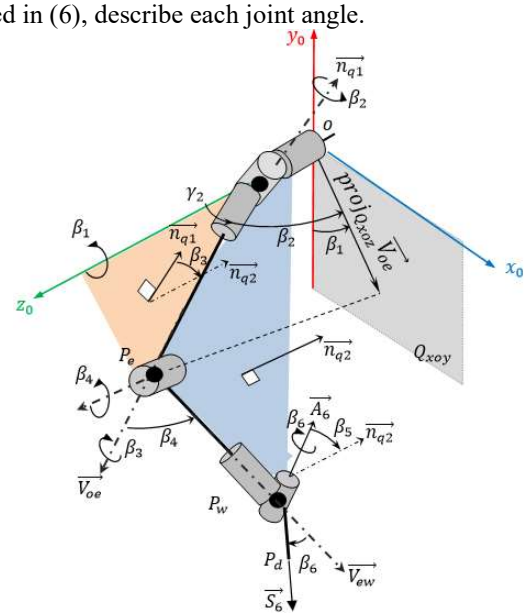


Fig.6: Vectorial description of arm's joint angles.

- *Joint angle* β_1

The vectors define the angle:

$proj_{Q_{xoz}}\overrightarrow{V_{oe}}$, $-\overrightarrow{y_0}$ and $\overrightarrow{z_0}$. Given that :

$$det_{\beta_1} = \det(-\overrightarrow{y_0}, proj_{Q_{xoz}}\overrightarrow{V_{oe}}, \overrightarrow{z_0})$$
$$cos(\beta_1) = -\overrightarrow{y_0}.proj_{Q_{xoz}}\overrightarrow{V_{oe}}$$

Hence

$$\beta_1 = a\tan2\left(det_{\beta_1}, cos(\beta_1)\right) \qquad (11)$$

- *Joint angle* β_2

From Fig.6, β_2 can be written as follows:

$$\beta_2 = \frac{\pi}{2} - \gamma_2$$

γ_2 is expressed by the formula bellow:

$$\gamma_2 = a\tan2\left(det_{\gamma_2}, \cos(\gamma_2)\right).$$

Where

γ_2 is the angle between $\overrightarrow{z_0}$ and $\overrightarrow{V_{oe}}$

$\overrightarrow{z_0}$ *and* $\overrightarrow{V_{oe}}$ are, respectivelly, the cosine vector of z_0-axis of fixed frame at schoulder and the vector between schoulder and elbow points.

$$\cos(\gamma_2) = \overrightarrow{z_0}.\overrightarrow{V_{oe}} \text{ and } det_{\gamma_2} = det\left[\overrightarrow{n_{q1}}\ \overrightarrow{z_0}\ \overrightarrow{V_{oe}}\right]^T$$

So

$$\beta_2 = \frac{\pi}{2} - a\tan2\left(det_{\gamma_2}, \cos(\gamma_2)\right) \quad (12)$$

- *Joint angle* β_3

The arguments for the angle β_3 are given bellow

$\cos(\beta_3) = \overrightarrow{n_{q1}}.\overrightarrow{n_{q2}}$ and $det_{\beta_3} = det\left[\overrightarrow{V_{eo}}\ \overrightarrow{n_{q1}}\ \overrightarrow{n_{q2}}\right]^T$.

So

$$\beta_3 = a\tan2\left(det_{\beta_3}, \cos(\beta_3)\right) \quad (13)$$

Where

$\overrightarrow{n_{q1}}$ and $\overrightarrow{n_{q2}}$ are respectivelly, the normal vectors of Q_1-plan and Q_2-plan. The first one is defined by two vectors $\overrightarrow{z_0}$ and $\overrightarrow{V_{oe}}$. The second one is defined by two vectors $\overrightarrow{V_{oe}}$ and $\overrightarrow{V_{ew}}$.

- *Joint angle* β_4

The joint angle β_4 is an angle between $\overrightarrow{V_{oe}}$ (arm) and $\overrightarrow{V_{ew}}$ (forearm) as illustrated in Fig.6. Where $\overrightarrow{n_{oe}}$ and $\overrightarrow{n_{ew}}$ respectively are the normalized vectors of $\overrightarrow{V_{oe}}$ and $\overrightarrow{V_{ew}}$.

The arguments for the angle β_4 are: $\cos(\beta_4) = \overrightarrow{n_{oe}}.\overrightarrow{n_{ew}}$ and $det_{\beta_4} = det\left[\overrightarrow{n_{q1}}\ \overrightarrow{n_{oe}}\ \overrightarrow{n_{ew}}\right]^T$

So

$$\beta_4 = a\tan2\left(det_{\beta_4}, cos(\beta_4)\right) \quad (14)$$

- *Joint angle* β_5

The joint angle β_5 is defined between the vectors $\overrightarrow{n_{q2}}$ and $\overrightarrow{A_6} = (a_x, a_y, a_z)^T$ of the z_6--axis as illustrated in Fig.6. Thus, β_5 is expressed in (15) as follows

$$\beta_5 = atan2\left(det_{\beta_5}, cos(\beta_5)\right) \quad (15)$$

Where

$$\cos(\beta_5) = \overrightarrow{n_{q2}}.\ \overrightarrow{A_6} \text{ and } det_{\beta_5} = det\left[\overrightarrow{n_{we}}\overrightarrow{n_{q2}}\overrightarrow{A_6}\right]^T.$$

- *Joint angle* β_6

The joint angle β_6 is the angle between the forearm and the hand of the robot. It is defined by the vectors $\overrightarrow{V_{ew}}$ (elbow-wrist) and $\overrightarrow{V_{wd}}$ (wrist-desired position) (Fig.6). Thus, β_6 is the angle between two vectors: $\overrightarrow{n_{ew}} = \frac{1}{|\overrightarrow{V_{ew}}|}\overrightarrow{V_{ew}}$ and $\overrightarrow{S_6} = (s_x, s_y, s_z)^T$. Consequently, the two arguments of β_6 are given below

$$\cos(\beta_6) = \overrightarrow{n_{ew}}.\ \overrightarrow{S_6} \text{ and } det_{\beta_6} = det\left[\overrightarrow{A_6}\overrightarrow{n_{ew}}\overrightarrow{S_6}\right]^T.$$

Finally, β_6 is expressed by (16)

$$\beta_6 = atan2\left(det_{\beta_6}, cos(\beta_6)\right) \quad (16)$$

4) Inverse Kinematic Modeling of the Lower Limb (Leg).

The leg, with 06 degrees of freedom (right or left), is composed of three links that are connected to each other (Fig.7). After calculating the position of the ankle and the knee, we proceed to identify the joint angles of this latter.

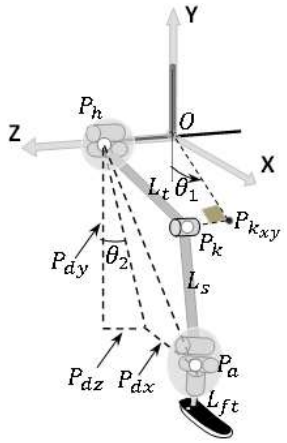


Fig.7: Geometric description of θ1 and θ2

- *Joint angle* θ1

The joint angle θ1 can be calculated via (17) using (3) with the help of the following three vectors: $\overrightarrow{OZ}$, $\overrightarrow{OY}$, $\overrightarrow{OP_{kxy}}$.

$$\theta_1 = a\tan 2\left(\det_{\theta_1},\ \cos\theta_1\right) \quad (17)$$

Where

$\det_{\theta_1} = \det(\overrightarrow{OZ}, \overrightarrow{OP_{kxy}}, -\overrightarrow{OY})$

$\cos\theta_1 = \cos(\overrightarrow{OP_{kxy}}, -\overrightarrow{OY})$

$\overrightarrow{OZ} = (0,0,1)$, $\overrightarrow{OY} = (0,1,0)$ two units vectors of the hip's frame.

$\overrightarrow{OP_{kxy}} = (P_{kx}, P_{ky}, 0)$ is the projection of knee's point on *xy-plane* that is defined by axes $\overrightarrow{OX}$ and $\overrightarrow{OY}$ of the hip's frame.

- *Joint angle* θ2

From Fig.7, these three points: P_h(hip's point) , P_k (knee's point) , and P_a (ankle's point) belong to the same plane which is initially parallel to *XY-plane* can move laterally only by the joint θ2.

Therefore, the joint angle θ2.can be calculated by (18) as follows

$$\theta_2 = \arccos\left(\frac{P_{az}}{P_{ay}}\right) \quad (18)$$

Where

$\left(P_{ax}, P_{ay}, P_{az}\right)^T$ are the coordinates of the vector $\overrightarrow{V_{oa}}$ that is defined between the origin of hip's frame and the ankle.

- *Joint angle* θ3

From Fig.8, the triangular formula bellow is defined

$$r_{ha}^2 = L_s^2 + L_l^2 - 2\,L_s\,L_l\cos\alpha$$

Where

$r_{ha} = P_{ax}^2 + P_{ay}^2 + P_{az}^2$

L_s: length of the shin.

L_l: length of the leg.

From the above equality, the angle α (between two links: leg and shin) is expressed in (19) bellow

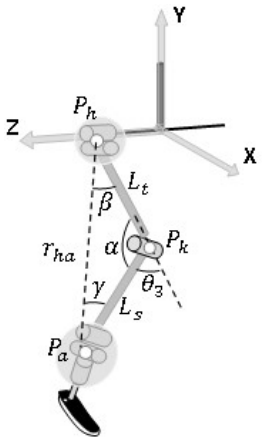


Fig.8: Geometric description of θ3.

$$\alpha = \arccos\left(\frac{r_{ha}^2 - L_{tb}^2 + L_J^2}{2\,L_{tb}\,L_J}\right) \qquad (19)$$

Therefore, from Fig.8 and using (19), θ3 is obtained through (20).

$$\theta_3 = \pi - \alpha \qquad (20)$$

- *Joint angle* θ4 *and* θ5

A new orthonormal frame $F_A(A, x_A, y_A, z_A)$ is defined at ankle point, where the position of L_f will be treated as a case of spherical coordinates by θ_4 and θ_5 (Fig.9).

Let P_Q be the plane by the normal vector $\overrightarrow{n_{ka}} = (a, b, c)^T$ expressed in (21)

$$P_Q: a\,x + b\,y + c\,z + D = 0 \qquad (21)$$

Where

$D = a\,P_{kx} + b\,P_{ky} + c\,P_{kz}$

And the constants a, b, c are the components of normal vector $\overrightarrow{n_{ka}}$ written in (22)

$$\overrightarrow{n_{ka}} = \frac{1}{\sqrt{V_{kax}^2 + V_{kay}^2 + V_{kaz}^2}}\left(V_{kax}, V_{kay}, V_{kaz}\right) \qquad (22)$$

Given that : $\overrightarrow{n_{ka}}$: Normal vector of the plane P_Q deduced from the vector $\overrightarrow{V_{ka}}$ defined by the points of the knee and the ankle.

Using (21) and (22), the axes of the new orthonormal frame $F_A(A, x_A, y_A, z_A)$ are defined bellow

$\overrightarrow{x_A} = (x1_A, x2_A, x3_A)$: $\overrightarrow{n_{ka}} = (a, b, c)^T$: Normal vector of the plane P_Q

$\overrightarrow{z_A} = (z1_A, z2_A, z3_A)$: $proj_{P_Q}\overrightarrow{Z_0}$: Projection of $\overrightarrow{Z_0}$ onto the plane P_Q

$\overrightarrow{y_A} = (y1_A, y2_A, y3_A)$: Cross product of $proj_{P_Q}\overrightarrow{Z_0}$ and $\overrightarrow{n_{ka}}$

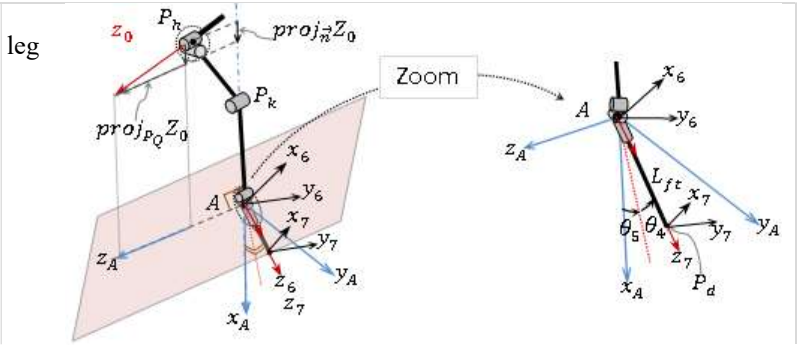


Fig.9: Geometric description of θ4 and θ5 into the new frame $R_A(A, x_A, y_A, z_A)$.

Conseauently, the new frame $F_A(A, x_A, y_A, z_A)$ can be described into the hip's frame by the following matrix.

$$T_{ha} = \begin{bmatrix} x_{A1} & y_{A1} & z_{A1} & P_{Ax} \\ x_{A2} & y_{A2} & z_{A2} & P_{Ay} \\ x_{A3} & y_{A3} & z_{A3} & P_{Az} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, one can express the determination of the coordinates of point Pd in the new frame FA as follows

$$^{R_A}P_d = \left[T_{ha}\right]^{-1}\,{}^{R_0}P_d \qquad (23)$$

Finally, from the coordinates $^{R_A}P_d$ given by (23), the determination of the rotational joints θ_4 and θ_5 is carried out using the spherical coordinates of the radius L_f, the foot link, (or $\boldsymbol{L_{ad}}$, the length between the ankle and the desired position).

$$\begin{cases} {}^{R_A}P_{dx} = L_f\cos\theta_4 \\ {}^{R_A}P_{dy} = L_f\sin\theta_4\cos\theta_5 \\ {}^{R_A}P_{dz} = L_f\sin\theta_4\sin\theta_5 \end{cases} \qquad (24)$$

Finally, from (24), θ_4 and θ_5 can be expressed by (25) and (26) bellow

$$\theta_4 = \arccos\left(\frac{^{R_A}P_{dx}}{L_P}\right) \qquad (25)$$

And

$$\theta_5 = \arctan\left(\frac{^{R_A}P_{dz}}{^{R_A}P_{dy}}\right) \qquad (26)$$

- *Joint angle* θ6

The last rotational joint θ_6 (See Fig.10) of the leg can be obtained through a simple matrix calculation given by (27).

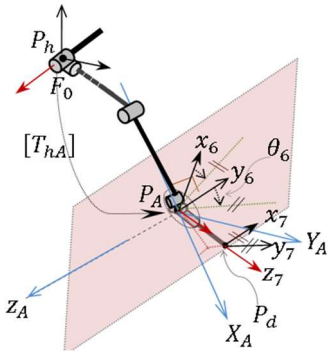


Fig.10: Geometric definition of θ_6 of the leg.

$$
\begin{aligned}
&{}^0T_7 = {}^0T_4\ {}^4T_5\ {}^5T_6\ {}^6T_7 \\
\Rightarrow\ &{}^6T_7 = \left[{}^5T_6\right]^{-1}\left[{}^4T_5\right]^{-1}\left[{}^0T_4\right]^{-1}\left[{}^0T_7\right]
\end{aligned}
\qquad (27)
$$

Where

${}^0T_7$: Matrix of the given situation of the end effector (foot position).

${}^0T_4$: Matrix of the situation of the frame $\boldsymbol{R_A}$ inserted at the robot's ankle. This matrix is none other than the matrix T_{ha}

$$[{}^0T_4] = [T_{hA}] = [{}^0T_1]\,[{}^1T_2]\,[{}^2T_3]\,[{}^3T_4]$$

${}^6T_7$: Transformation matrix between R_6 and R_7 which is expressed by θ_6.

The components of the matrix ${}^6T_7$ are given below:

$$
{}^6T_7 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & L_P \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Thus, (27) can be reformulated as follows:

$$
{}^6T_7 = \left[{}^5T_6\right]^{-1}\left[{}^4T_5\right]^{-1}\left[\ T_{ha}\right]^{-1}\left[{}^0T_7\right] \qquad (28)
$$

By identifying the terms in (28), θ_6 can be easily found in (29).

$$
\theta_6 = a\tan 2\left(\sin\theta_6, \cos\theta_6\right) \qquad (29)
$$

Note.

Due to symmetry of robot, the same modeling procedure used for the right leg will be applied to the left leg.

IV. Parametrics Equations of End-Effector's Trajectories

In order to verify the obtained results of the used approach in solving IK problem of biped robot, we plan to conduct simulations using Matlab software and leveraging the solutions the previously obtained results. However, it is necessary to determine the set of trajectories for the various members of the robot.

A. The trajectory of the hip.

The trajectory of the hip is defined by the three parametric equations below: (30), (31) and (32)

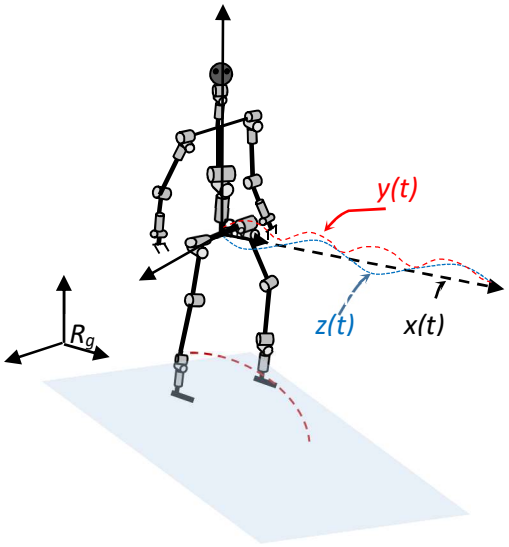


Fig.11: Trajectory hodograph of the hip.

$\boldsymbol{x_h(t)}$: Linear equation on x-axis.

$$
x_h(t) = at + b \qquad (30)
$$

$\boldsymbol{y_h(t)}$: Polynomial equation of degree five in the plane (o:x,y).

$$
y_{hs}(t) = k_0 + k_1 t + k_2 t^2 + k_3 t^3 + k_4 t^4 + k_5 t^5 \qquad (31)
$$

$\boldsymbol{z_h(t)}$: Sinusoidal equation in the plane (o: x, z)

$$
z(t) = A_z \sin\left(a\left(x(t) - x_0\right)\right) \qquad (32)
$$

B. Trajectory of the foot in flight.

Let $X_{pS}(t)$ and $X_{pd}(t)$ denote the functions expressing respectively the trajectories of the simple support Phase (SSP) and the double support phase (DSP). Given that.

$X_{PS}(t) : \left\{ x_{ps}(t), y_{ps}(t), z_{ps}(t) \right\}$ and

$X_{Pd}(t) : \left\{ x_{pd}(t), y_{pd}(t), z_{pd}(t) \right\}$

Where

$\left\{x_{ps}(t), y_{ps}(t), z_{ps}(t)\right\}$ and $\left\{x_{pd}(t), y_{pd}(t), z_{pd}(t)\right\}$ denote its parametric equations expressed in the principal frame. However, the double support phase represents 20% of the single support phase [9].

$\boldsymbol{x_h(t)}$: Cubic polynomial equation.

$$
x_{ps}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \qquad (33)
$$

$\boldsymbol{y_h(t)}$: Quintic polynomial equation.

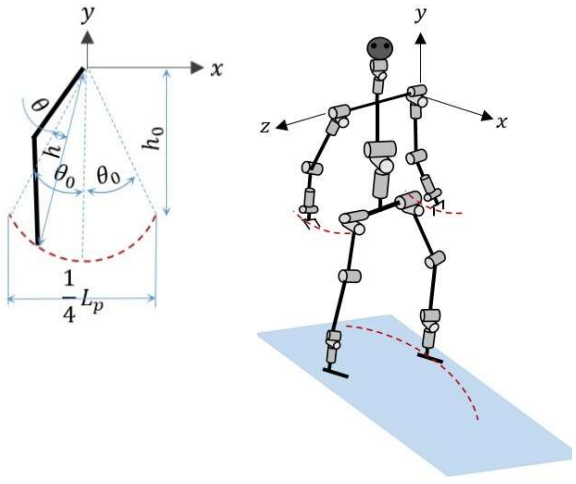
$$
y_{ps}(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \qquad (34)
$$

$z_h(t)$: no movement of the foot along the z-axis.

$$z_p(t) = 0 \tag{35}$$

C. Trajectory of the upper limbs.

The two upper limbs swing in an alternating and opposite manner between a forward and backward movement. (Fig. 12).



a b

Fig.12: Hodograph of the robot's upper limbs.

Their parametric equations with respect to the shoulder frame are expressed by the following equations

$x_m(t)$: Linear equation along the x-axis.

$$x_m(t) = at + b \tag{36}$$

$y_m(t)$: Cosinusoidal equation

$$y_m(t) = -R\cos\theta(t) \tag{37}$$

Where:

$R \le L_a + L_{fa}$

$\theta(t) = \frac{2}{T_c}\theta_0\, t - \theta_0$: Linear equation.

$z_h(t)$: No movement of the foot along the z-axis.

$$z_m(t) = 0 \tag{38}$$

Note.

All the constants in the parametrics equations (30-38) of trajectories are detrmined with respect to condition and exigences of walk of robot. Primarlly, bellow parameters are used:

L_p : Step length.

T_s : Step period for the SSP.

T_d : Step period for the DSP.

H_f : Maximum height of the flight foot.

S_f : Corresponding abscissa to H_f .

H_{max} and H_{min} : Respectively, maximum and minimum height of the hip .

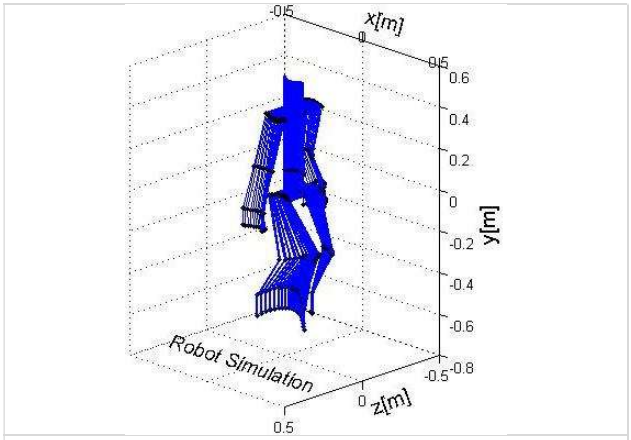
H_{x_0} : Hip initial position on *x-axis*.

V. Simulation of Inverse Kinematic Model of the Biped Robot

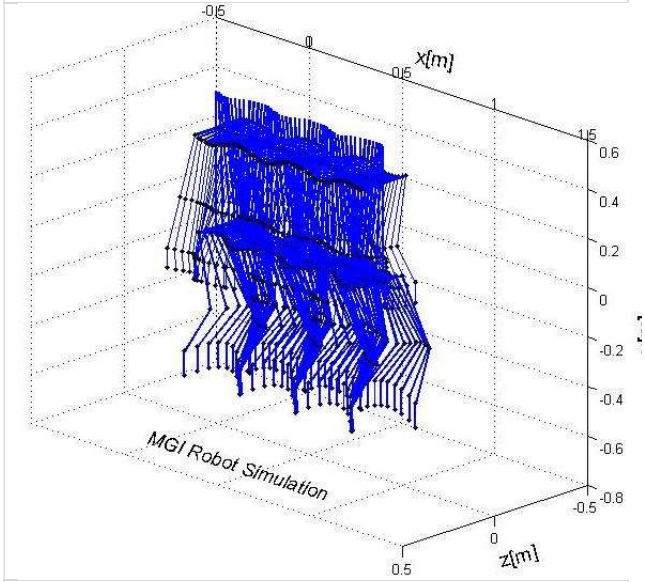
The simulations are conducted on the bipedal robot to validate the IKM solutions obtained previously. The bipedal robot should walk correctly by following the trajectories imposed on its gait-members (hip, upper limbs, and lower limbs).

Simulation parameters values.

- Lengths of links of robot are given in Tab.1.
- Maximum and minimum height of the hip respectively : $h_{max} = 0.69$; $h_{min} = 0.66[m]$
- Maximum foot height in flight: $H_f = 0.10[m]$.
- One-step period : T_p=Ts+Td =1.3+0.3 =1.6[s]
- Average time corresponding to H_f : $T_m = T_s / 2$[s]
- Step length: $L_p = 0.32[m]$.
- Final position of the hip in DSP : $H_{xf} = 0.5.Lp + H_{x_0}.\ [m]$



a



b

Fig.13: Simulation of the inverse kinematic model of the robot (IKM). a. simulation of a single step. b. simulation of six successive steps.

These simulations, depicted in Fig.13, demonstrates that the developed solutions in solving IKM presented above is correct and accurate.

VI. General Conclusion

An attractive and basic topic in robotics was developped in this work. An application on biped robot with 30 degree of freedom was presented. Topologically, this robot consists of a central mechanism (the trunk), two upper limbs (right arm and left

arm), two lower limbs (right leg and left leg), and a neck-head mechanism.

This work extends to inverse kinematic modeling by employing an alternative approach based on vectors and the geometric environment of the robot. This approach offers a significant advantage in obtaining solutions through linear and uncoupled equations. In fact, all joint variables of the bipedal robot with 30 degrees of freedom are determined using this approach. Simulations were conducted and the obtained results align with the expected goals. Thus, this work contributes to the advancement of knowledge in the field of robotics.

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