Fractional-order PD control of a parallel Delta robot: Experimental results

Someya Amrane, Chems Eddine Boudjedir and Djamel Boukhetala

Abstract− In this paper, a fractional-order proportional–derivative (PD) controller is proposed as a means to enhance the trajectory tracking performance of a parallel Delta robot. The highly coupled and nonlinear dynamics of the Delta robot pose significant challenges for conventional integer-order PD controllers, often resulting in limited tracking accuracy. To address these limitations, the integer-order derivative term is replaced by a fractional-order derivative, thereby providing additional tuning flexibility and improved dynamic behavior. In experimental studies, a comparison is conducted between the fractional-order and integer-order PD approaches, as well as an evaluation of the influence that different fractional derivative orders have on robot tracking performance.

***Keywords*− Fractional-order, PD controller, Trajectory tracking, Delta robot.**

I. Introduction

Many theoretical contributions to fractional calculus have been proposed by Euler, Liouville, Riemann, and Grünwald [1]. These definitions have been successfully applied in multiple domains, such as electromagnetism and electrochemistry. For a historical introduction to fractional calculus, the reader can refer to [2].

The application of fractional calculus has experienced significant growth over the last decades, due to its robustness and improved tracking performance. Fractional calculus has been applied in many engineering fields, such as robotics [3], autonomous underwater vehicles [4], and wind turbine generators [5].

Parallel kinematic robots offer several advantages over serial robots, such as high rigidity, accuracy, and load capacity. Professor Raymond Clavel invented the parallel Delta robot as an efficient solution for repetitive pick-and-place operations. The original prototype features three translational degrees of freedom and one rotational degree of freedom [6]. The reader may refer to the survey [7] for further designs of the Delta robot.

The robot manipulator is commonly controlled using conventional PID controllers [8]. However, this control law is often inadequate for applications requiring high precision under fast dynamic motions, due to the fact that the PD control parameters are chosen without fully considering the coupling effects. To overcome this issue and improve trajectory tracking performance, many works have been proposed, such as nonlinear PD control [9], iterative learning control [10-12], time delay control [13], and sliding mode control [14].

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S.Amrane and D.Boukhetala are affiliated with the laboratory of process control, Ecole nationale Polytechnique, Algiers, Algeria (someya.amrane@g.enp.edu.dz; djamel.boukhetala@g.enp.edu.dz)
C.Boudjedir is affiliated with LRPE laboratory, University of science and technology Houari Boumediene, Algiers, ALGERIA (chemseddine.boudjedir@usthb.edu.dz)

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In recent years, both fractional calculus and model-free control strategies have attracted considerable attention. Several control frameworks have incorporated fractional-order operators into the control loop, for instance, fractional adaptive control [15] and robust control design CRONE [16].

Fractional-order PID controllers have demonstrated superior robustness and performance compared to conventional PID controllers. Podlubny's proposal introduced a generalized fractional-order PID controller of the form $PI^{\lambda}D^{\mu}$, where λ and μ are non-integer orders. By appropriately tuning these parameters, the tracking accuracy can be significantly improved [1].

Fractional-order PID controllers have been further investigated in [17], where the robustness and performance were enhanced. In [18], the fractional-order PID controller was applied to a planar parallel robot to improve trajectory tracking accuracy. In [19], a fractional-order PID was designed to control a parallel robot, resulting in reduced tracking error and eliminated overshoot.

The main contribution of this paper is the design and experimental implementation of a fractional-order PD controller for trajectory tracking of a parallel Delta robot. Experimental studies are conducted to evaluate the effectiveness of the proposed approach. A comparative analysis between the fractional-order PD controller and the conventional integer-order PD controller is also conducted.

The remaining sections of this paper are organized as follows: section II introduces the dynamic model of the Delta robot. Section III presents the controller design. While in section IV, experimental results are presented. Finally, section V provides some conclusions.

II. Dynamic model

The Delta robot shown in Fig. 1 is equipped with three kinematic chains, each consisting of a servo motor and a reducer connected to the upper arm. The forearm of the Delta robot is linked to both the upper arm and the travelling plate.

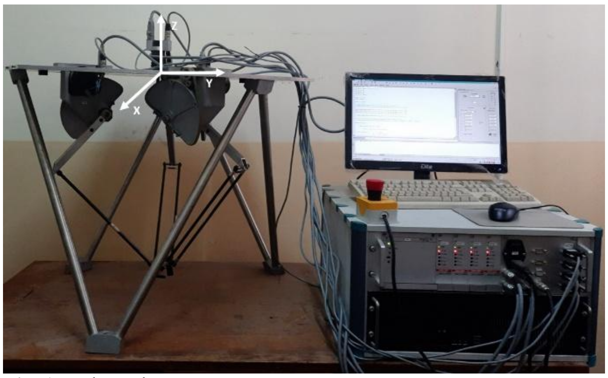

Fig. 1: Delta Robot

The robot dynamics is described as in [11]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \quad (1)$$

Where:

$$M(q) = I_b + m_{nt}J^T J$$
$$C(q,\dot{q}) = J^T m_{nt}\dot{J}$$
$$G(q) = -\tau_{G_n} - \tau_{G_b}$$

The generalized joint vector is denoted as $q = [q_1, q_2, q_3]^T$, the inertia matrix is represented by M(q) $\in R^{3\times3}$, the vector resulting from centrifugal and Coriolis forces is denoted as $C(q,\dot{q})\dot{q} \in R^{3\times3}$. $G(q) \in R^{3\times1}$ refers to the gravitational vector. τ, τ_{G_b} and τ_{G_n} represent, respectively, the joint torque, the torque produced by the gravitational force of the arms and the torque produced by the inertial force. The Jacobian matrix is denoted as J, and its derivative respect to time is given as $\dot{J}$.
m_{nt} signifies the total mass, which includes the mass of the travelling plate, the playload mass and the combined masses of the three forearms.

The expression of the torques is given as follows:

$$\tau_{G_n} = J^T m_{nt}[0 \quad 0 - g]^T \quad (3)$$
$$\tau_{G_b} = m_b r_{G_b} g[\cos q1 \quad \cos q2 \quad \cos q3]^T \quad (4)$$

The detailed expressions of J, $\dot{J}$, m_{nt} and r_{G_b} are given in [11]. Table I describes the parameters of the robot.

Table. I
GEOMETRIC AND DYNAMIC PARAMETERS

Parameters	DESCRIPTION	Value
$\mathbf{L_a}$	Upper arm length	0.380 m
$\mathbf{L_b}$	Forearm length	0.205 m
$\boldsymbol{m_n}$	Traveling plate mass	0.42 kg
$\mathbf{m_{br}}$	Upper arm mass	0.098 Kg
$\mathbf{m_{ab}}$	Forearms masses	0.028 Kg
$\mathbf{m_c}$	Elbow mass	0.016 Kg

III. CONTROLLER DESIGN

The PD controller is proposed in joint space as follows:

$$\tau = k_{p0}\tilde{q}(t) + k_{d0}\dot{\tilde{q}}(t) \quad (5)$$

In which, k_{p0} and k_{d0} are constant diagonal matrices. $q(t)$ and $\dot{q}(t)$ are given as follows:

$$\tilde{q}(t) = q_d(t) - q_k(t)$$
$$\dot{\tilde{q}}(t) = \dot{q_d}(t) - \dot{q_k}(t)$$

Where $q_d(t)$ and $\dot{q_d}(t)$ represent the desired joint position and the desired joint velocity, respectively. The actual joint position and the actual joint velocity are denoted as $q_k(t)$ and $\dot{q_k}(t)$ respectively.

Since conventional PID controllers may not achieve satisfactory performance for tasks requiring high precision, many studies have applied fractional-order PID controllers to improve accuracy and trajectory tracking. In this paper, a fractional-order PD controller is implemented on the Delta robot. The control law includes three parameters: the proportional gain K_p, the derivative gain K_d, and the derivative fractional-order μ.

By introducing the fractional derivative order, the controller can achieve a satisfactory trade-off among the advantages and drawbacks of the conventional PD controller, such as enhanced stability provided by the derivative term, while mitigating its main disadvantage, i.e., high sensitivity to noise.

The continuous differential operator is given by :

$${}_aD_t^\mu = \frac{d^\mu}{dt^\mu} \qquad \mu > 0$$

Where, $\mu \in R$ is the operation order.

Grunwald-Letnikov definition is given by :

$${}_aD_t^\mu f(t) = \frac{{}_a d_t^\mu f(t)}{dt^\mu} = \lim_{h\to0}\left\{\frac{1}{h^\mu}\sum_{k=0}^{\frac{t-a}{h}}(-1)^k\binom{\mu}{k}f(t-kh)\right\}$$

The fractional-order derivative of the function f requires knowledge of $f(t)$ over the interval [a, t], in contrast to the integer order which only requires knowledge of f near t. This feature leads to the conclusion that fractional-order systems are long-memory systems.

The fractional-order PD controller is expressed as:

$$\tau = k_p\tilde{q}(t) + k_d D^\mu\tilde{q}(t) \quad (8)$$

where the torque τ represents the control signal.

Fractional-order functions must approximated by integer-order expressions to be easily handled during software implementation. The numerical approximation for fractional calculus used in this paper is the Grünwald–Letnikov method, based on the Taylor expansion [1] :

$${}_{\left(k-\frac{L}{h}\right)}D_{t_k}^\mu\ \tilde{q}(t) \approx h^{-\mu}\sum_{j=0}^{k}(-1)^j\binom{\mu}{j}\tilde{q}(t_{k-j})$$
$$= h^{-\mu}\sum_{j=0}^{k} c_j^{(\mu)}\,\tilde{q}(t_{k-j}) \quad (9)$$

In which, $t_k = kh$, h is the sampling period, and L represents the memory length,

The binomial coefficients can be calculated as follows:

$$c_j^{(\mu)} = \left(1 - \frac{1+\mu}{j}\right)c_{j-1}^{(\mu)} \quad (10)$$

and $c_0^{(\mu)} = 1$

The scheme of the fractional-order PD controller is shown in Fig. 2, where IGM illustrates the inverse geometric model and x_d denotes the desired trajectory in the operational space.

Remark 1: The fractional-order PD controller can be widely used in industrial applications due to its ease of implementation.
Remark 2: Unlike the control strategies which require an exact mathematical model, the fractional-order PD controller is model-free.

Remark 3: The parameters of the fractional-order PD controller allow achieving a better trade-off between the positive and negative effects of the derivative action.

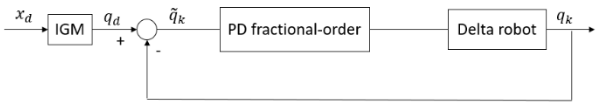

Fig. 2: Scheme of the proposed controller.

IV. Experimental Results

The experimental results obtained by applying the fractional-order PD control law (8) on the Delta robot of Fig. 1 are presented in this section.

The robot utilizes brushed DC motors with a belt-driven transmission having a ratio of r=12. The operational trajectory is executed with a maximum acceleration of 15 m/s^2 [23, 24]. The data were collected by sampling at 1 kHz, and the control algorithms were implemented in C language.

The tracking performance evaluation involves the utilization of the Maximum Absolute Error (MAE) and Root Mean Square Error (RMSE) as criteria. The expressions of these criteria are as follows:

$$RMSE_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - x_{d_i})^2} \quad (11)$$

$$MAE_x = \max\left(|x_i - x_{d_i}|\right) \quad (12)$$

Where n represents the number of samples, x_d represents the desired trajectory in the operational space and x_i is the actual response in operational space.

The expression of the RMSE and MAE when considering all the three axes is given by:

$$RMSE = \sqrt{RMSE_x^2 + RMSE_y^2 + RMSE_z^2} \quad (13)$$

$$MAE = \max\left(MAE_x, MAE_y, MAE_z\right) \quad (14)$$

The controller gains and fractional-order are selected as $k_p = diag\{2.2, 2.2, 2.2\}$, $k_d = diag\{0.0145, 0.0145, 0.0145\}$ and $\mu = 1.12$. These parameters are obtained using an iterative tuning procedure, in which tracking performance is evaluated by varying one parameter at a time while holding the others constant. The parameter k_p is maintained at a moderate level to highlight the impact of the order μ. Increasing k_p results in a faster response; however, it may also amplify overshoot, particularly when combined with a high μ.

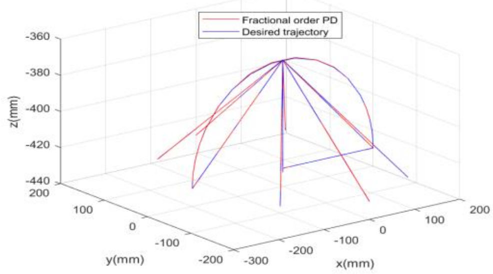

Fig. 3: The operational trajectory tracking under the fractional-order PD controller

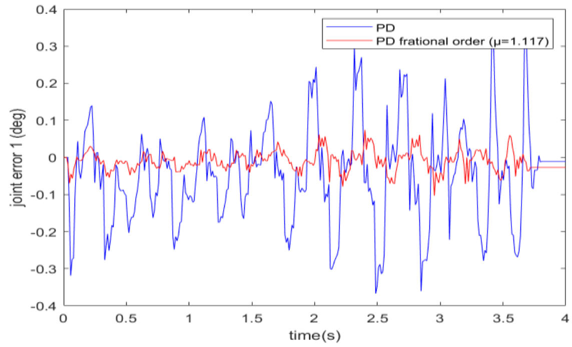

Fig. 4: Experimental tracking error of joint 1.

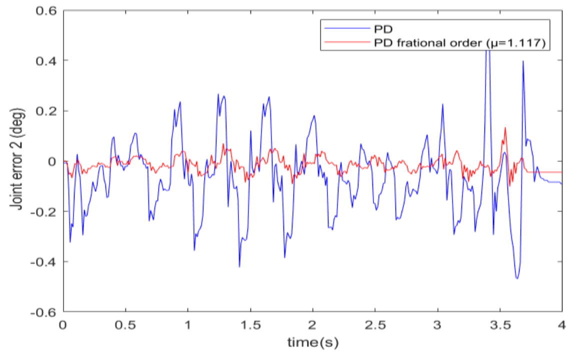

Fig. 5: Experimental tracking error of joint 2.

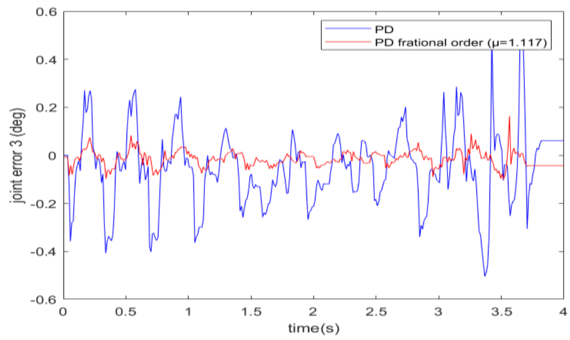

Fig. 6: Experimental tracking error of joint 3.

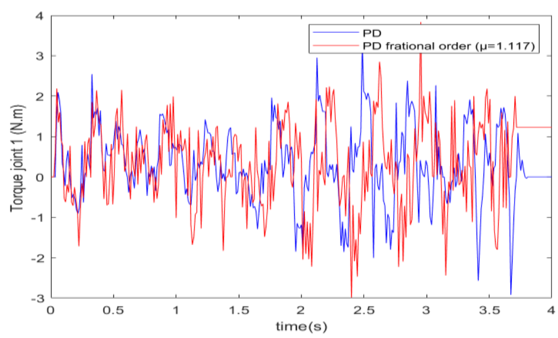

Fig. 7: Control torque of joint 1.

Fig 3 illustrates the trajectory tracking in the operational space under the proposed fractional-order PD controller. Figures 4, 5, and 6 depict the tracking error under the proposed fractional-order PD and PD controller of joints 1, 2, and 3, respectively. The figures point out that the tracking error under the fractional-order PD is inferior compared to the error under the PD controller. The proposed controller exhibits an RMS error equal to 0.17 mm, which is less than 80.4% of that provided by the PD controller. For MAE, the fractional PD controller can ensure 0.49 mm, which is less than 80.8% of that ensured by the PD controller.

Figures 7, 8, and 9 show the control torque signals under the PD and fractional-order PD controller for joints 1, 2, and 3, respectively. It is observed that the torque signals have nearly the same amplitude and variation. Table II presents the tracking performance of both controllers.

Figs. 10 to 15 represent the tracking errors and control torque signals under the fractional-order PD controller for different derivative fractional-orders of joints 1, 2, and 3, respectively. It can be observed that when μ=0.91, the tracking error is much larger compared to the cases when μ=1.11 or μ=1.21. The RMSE decreases from 0.43 mm at μ=0.91 to 0.17 mm at μ=1.11, and to 0.11 mm for μ=1.21.

Nevertheless, increasing the value of the fractional order μ leads to significant oscillations in the control signals, which may degrade the tracking performance of the proposed controller.

Table III outlines the RMSE and MAE of the fractional-order PD controller under different fraction orders.

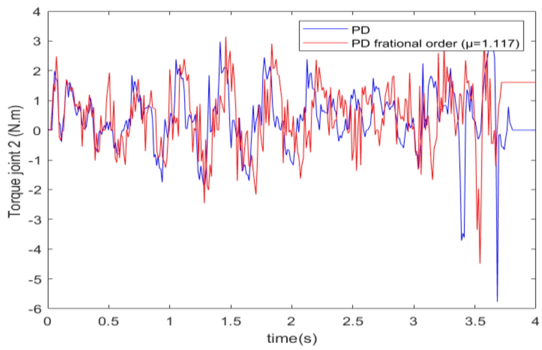

Fig. 8: Control torque of joint 2.

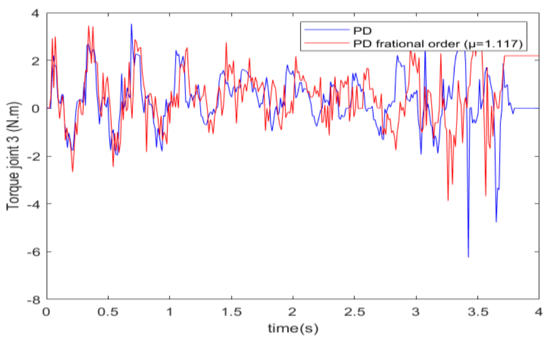

Fig. 9: Control torque of joint 3.

Table. II
TRACKING PERFORMANCE

Performance	PD	Fractional-order PD
RMSE (mm)	0.87	0.17
MAE (mm)	2.56	0.49

Table. III
TRACKING PERFORMANCE FOR DIFFERENT FRACTIONAL ORDERS

Performance	μ = 1.21	μ = 0.91	μ = 1.11
RMSE (mm)	0.11	0.87	0.17
MAE (mm)	0.44	2.56	0.49

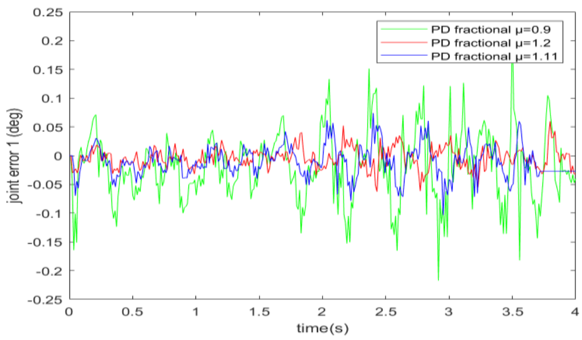

Fig. 10: Tracking error for different fractional derivative order of joint 1.

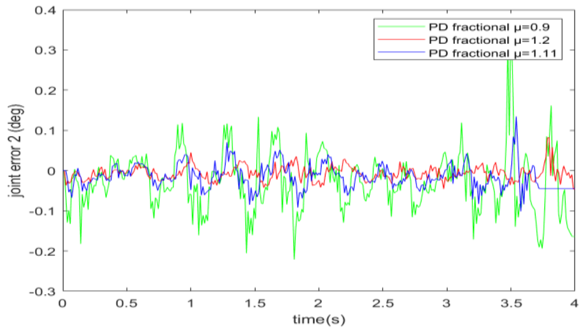

Fig. 11: Tracking error for different fractional derivative order of joint 2.

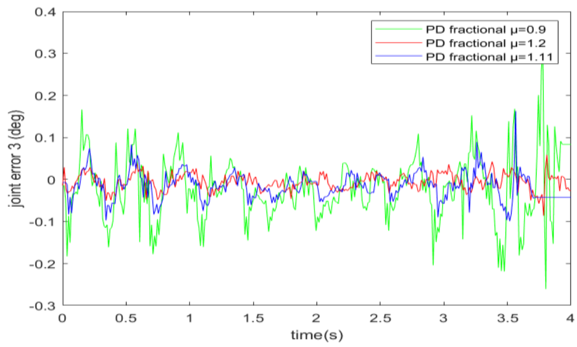

Fig. 12: Tracking error for different fractional derivative order of joint 3.

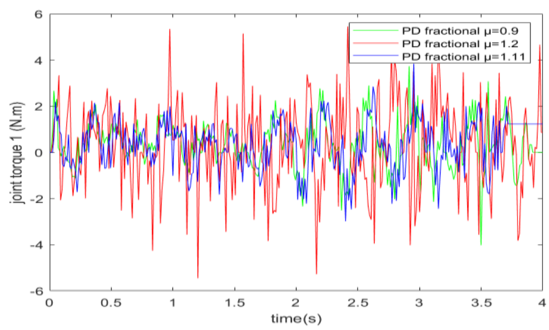

Fig. 13: Control torque for different fractional derivative order of joint 1.

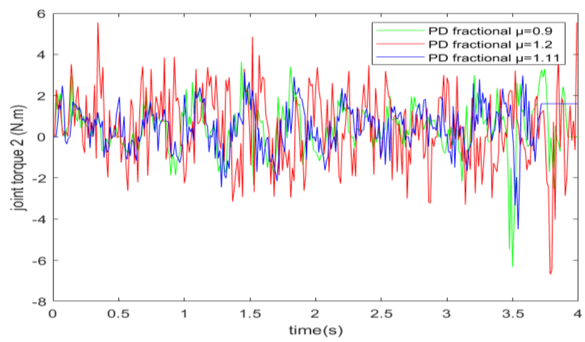

Fig. 14: Control torque for different fractional derivative order of joint 2.

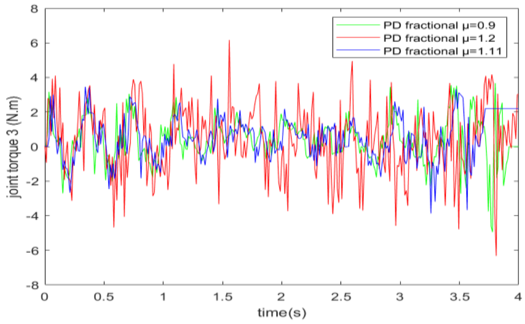

Fig. 15: Control torque for different fractional derivative order of joint 3.

V. CONCLUSION

In this paper, a model-free fractional-order PD controller is applied to a Delta robot to address the trajectory tracking problem. To improve the performance of the conventional PD controller, a fractional derivative order is introduced. Experimental studies on the Delta robot demonstrate the effectiveness of the proposed approach. The results show that the fractional-order PD controller achieves better tracking performance compared to the conventional PD controller, while both controllers exhibit similar control torques. It is also found that the fractional derivative order μ must be carefully chosen to balance tracking accuracy and smoothness of the control torque.

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Someya Amrane received the M.Sc. degree in control engineering from the École Nationale Polytechnique, Constantine, Algeria, in 2019. She is currently working toward the Ph.D. degree in the École Nationale Polytechnique, Algiers, Algeria. Her research interests include fractional-order control, sliding mode control, time delay control, and robotics.

Chems Eddine Boudjedir received the M.Sc. and Ph.D. degrees in control engineering from the École Nationale Polytechnique, Algiers, Algeria, in 2015 and 2019, respectively. He is currently an Associate Professor with the University of Science and Technology Houari Boumediene, Algiers. His research interests include iterative learning control, sliding mode control, time delay control, disturbance observer, and robotics.

Djamel Boukhetala graduated in automation in 1989 and received the Magister and Ph.D. degrees (Doctorat d'État) in automatic control from the École Nationale Polytechnique (ENP), Algiers, Algeria, in 1993 and 2002, respectively. From 1996 to 1999, he was the Head of the Department of Automatic Control. He was the Director of the Control Process Laboratory from 2005 to 2013 and the Director of Postgraduate Studies and Scientific Research at ENP from 2010 to 2019. He is currently a Full Professor in automatic control with ENP. He is currently a member of several scientific councils in higher education and scientific research institutions. His research interests are decentralized control, nonlinear control, fuzzy control, and artificial neural network control applied to robotics, smartgrids, and industrial processes.