Modal identification and dynamic analysis of a 1000 years old historic minaret of Kalaa Beni-Hammad

Nouredine Bourahla, Zakaria Assameur, Mohamed Abed, and Ahmed Mébarki.

Abstract− From structural and material point of view each historic monument is a textbook case, as it differs in shape, age, material characteristics and level of preservation. New means and approaches are used for geometric and shape survey, material and modal characterization, which helps restitute detailed information regarding the structural elements of the edifices. This paper investigates a 1000 years old historic minaret having a square base of 6.50 m of side and 24.70 m height. An ambient vibration survey has been carried out and the results were used to update a Finite Element (FE) model of the minaret in two stages. First, the elastic modulus of the rubble stone masonry has been determined by matching the experimental and numerical fundamental frequency. In a second phase, a frequency response function (FRF) correlation is carried out iteratively by adjusting the damping ratios to minimize the FRF error. The elastic modulus of the rubble stone masonry is found to be in the lowest range of values given in the literature, which confirms a deteriorated quality of the material. A seismic performance of the structure has been carried out using the updated FE model subjected to a set of acceleration time histories having the mean spectrum matching a compatible site spectrum. The stress concentration zones have been determined and a steel tie system to retrofit the structure is proposed. Beside the proposed investigation framework, the characterization results are valuable facts that can be added to the available database of historic monuments.

Keywords− **Historic monument, Kalaa Beni-Hammad, قلعة بني حماد, Model updating, Ambient vibration testing, Structural strengthening, Finite element method model calibration.**

NOMENCLATURE

The main abbreviations used in this article are listed below:

AVT	Ambient Vibration Testing.
FDAC	Frequency Domain Assurance Criteria.
FE	Finite Element.
FRF	Frequency Response Function.
MAC	Modal Assurance Criteria.
PPM	Pick-Picking Method

I. INTRODUCTION

The historic heritage is a universal valuable asset that requires more attention to preserve. Those edifices, located in seismic prone regions, have survived several earthquakes and are still standing. They can be of particular interest from a structural engineering point of view. The research activity for structural assessment [5], material properties evaluation [4] and retrofitting of historic buildings all over the world continues to be very attractive and energetic [2, 9, 17, 19]. This has led to development of a variety of methods and practices for diagnosis and seismic strengthening of ancient monuments where advanced technological means are used to scan, test and model the edifices [11, 15]. The use of such innovative methods enhances the precision of the analyses, but uncertainties in the material properties are still a challenge and vary significantly from case to case. More data on different types of historic monuments with different materials and different ages is still needed to understand better their structural behaviors. In this paper, a 1000 years old minaret of about 24 m height made of rubble stone masonry is experimentally and numerically investigated. The measured and the numerical FRF curves were used to match the natural frequencies of the structure and their corresponding damping ratios. A conceptual strengthening solution is proposed and the performance is assessed.

II. HISTORICAL BACKGROUND AND STRUCTURAL IDENTIFICATION AND MODELING OF THE MINARET

Founded in 1007 by Hammad ibn Bologhine [10], the fortress (Kalaa) of Beni-Hammad has played an important role as a capital of North Africa during the eleventh century. In 1980, UNESCO inscribed the site as a World Heritage Site. The archeological site is located on the Hodna plain at 36 km to the northeast of the town of M'Sila (Algeria). The region is classified as moderate seismic zone IIa by the Algerian code RPA99 v2003 [20].

The historic monument object of this study is a remaining minaret of a great mosque in ruin, having a square base of 6.50 m of side and 24.70 m height. Structurally, the edifice is made of a 1.80 m x 1.80 m square core surrounded by a 1.20 m thick walls both supporting the stairs and the barrel-vaulted ceilings creating a monolithic ensemble (Fig. 1). All the structural elements are made of local sandstone ruble, which has suffered severe ageing and environmental actions. The upper part and the ornament of the minaret tumbled down long time ago at an

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undefined age [1]. The edifice has been restored according to a plan of protection and restoration of the site set up by UNESCO in the period of 1976-82 [20].

For the purpose of the present study, a simple Finite Element (FE) model is elaborated. The core of the minaret is modeled using solid elements. The outer walls, the stairs and ceilings are modeled using thick shell elements. An adequate meshing is adopted to meet the required precision without oversizing the model. The structure is assumed to be fixed at its base. The specific weight of the material is obtained from the literature for similar type of masonry and is taken equal to 18 kN/m^3 with a Poisson ratio equal to 0.24 [18].

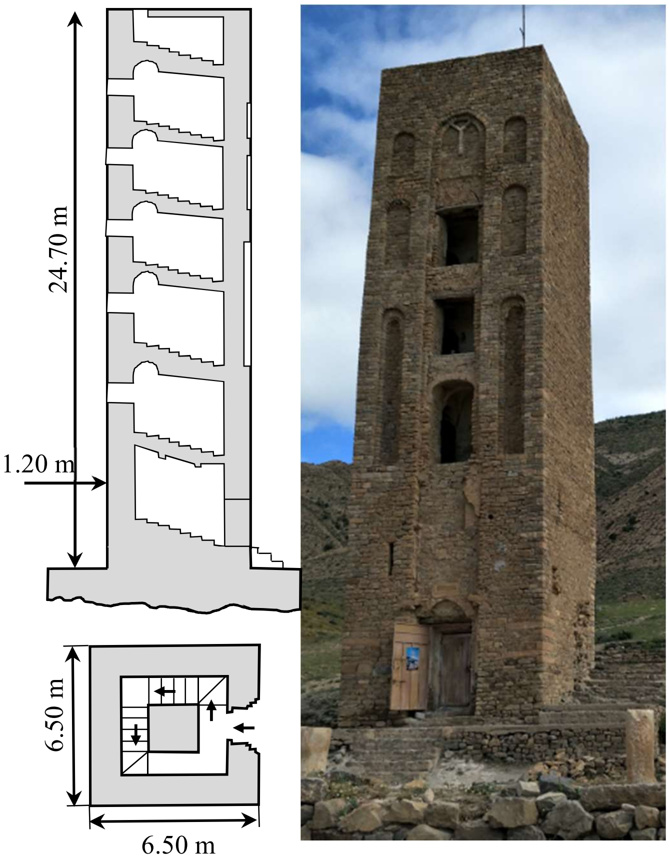

Fig. 1: Global view, layout and elevation sections of Kalaa beni-Hammad.

III. AMBIENT VIBRATION TESTING (AVT) AND MODEL UPDATING

Ambient vibration method has been used for more than five decades for full scale testing of civil engineering structures [13]. Its simplicity, cost effectiveness and wide range of applicability made of it a powerful tool for in-situ identification testing. One of the most frequent uses of AVT involves identification of natural frequencies, mode shapes of vibration and equivalent viscous damping ratio of full-scale structures [6, 21, 24]. These results are used to update the FE model by matching the experimental and numerical modal properties [7, 14, 22]. First, a careful selection of the model parameters to be modified by the updating procedure is to be made in order to ensure that the necessary changes to the model are realistic and physically realizable and meaningful [14]. In this instance, the material properties, which are uncertain, are chosen as matching parameters as the geometric characteristics and masses are precisely determined.

A. *Test set-up and procedure*

The tests were performed using three degrees of freedom seismometer type Lennartz electronic (Le3Dlite) and a data acquisition system type City Shark II. The measured signals were processed using the GEOPSY program [25] capable to perform most of the signal processing operations for the analysis of ambient vibration data. Seven measurement points were located along the height of the minaret at the façade side to record the vibration along two horizontal and one vertical axes (Fig. 2). Two other recording points were used; one point was positioned at the corner of the roof and the other one at a free field. The recording time for each sequence was set to 6 mn and found to be largely sufficient to obtain smooth frequency response function (FRF) curves. Measurement locations were chosen to capture the fundamental and second lateral and torsional modes.

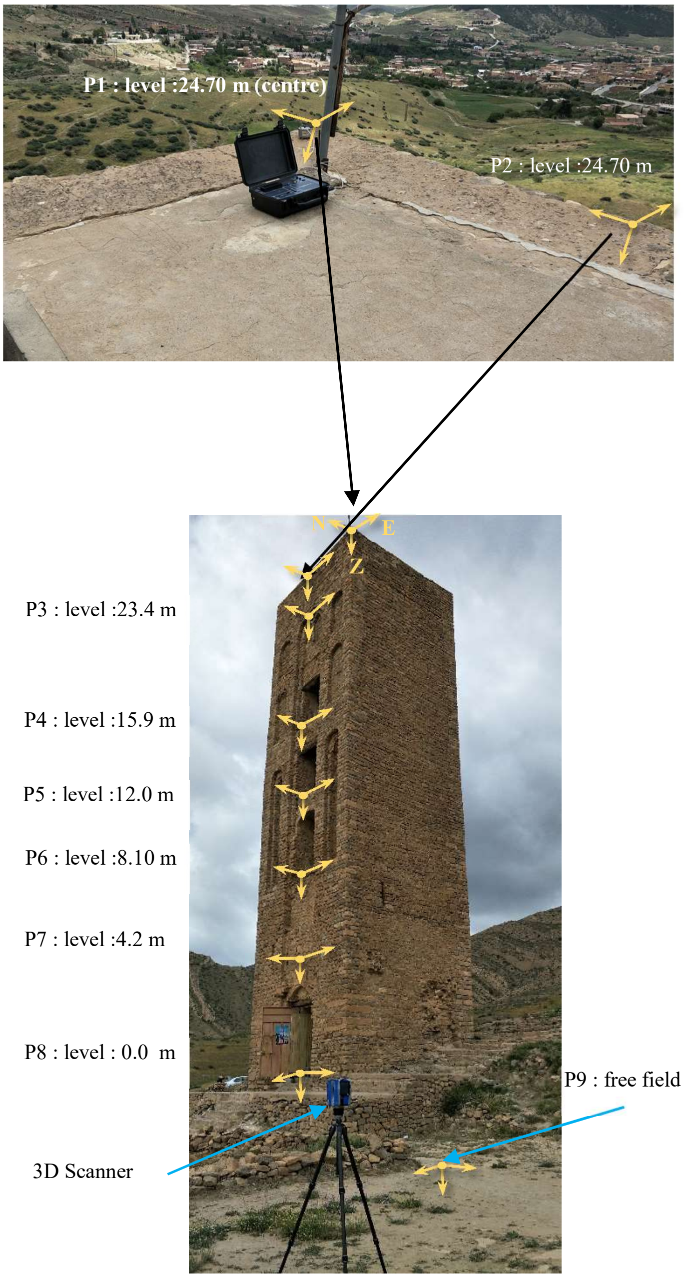

Fig. 2: Global view of the minaret with the positions of the measurement points.

B. *Frequencies identification*

The natural frequencies of Kalaa Beni-Hammad minaret were identified by applying the peak-picking method (PPM) on the FRFs. The curve in Fig. 3 shows the FRF of the vibrations measured along the two horizontal axes denoted as E-axis and

N-axis at the edge of the floor corresponding to point 1. The clearly distinct first peaks at 1.88 Hz and 1.90 Hz correspond to the fundamental lateral mode along N and E directions respectively (Fig 4). It should be noted that the peak corresponding to the torsional mode is missing on the FRF curve of record point 2, which is located at mid-façade on the N-axis where only lateral modes in N-direction are detectable.

The amplitudes of fundamental modes dominate those of the higher modes. The latter are more apparent at lower heights. As shown in Fig. 5, the peaks of the second lateral modes are comparable to those of the fundamental modes. A peak at frequency of 10.83 Hz corresponds to the pure vertical mode (Fig. 6).

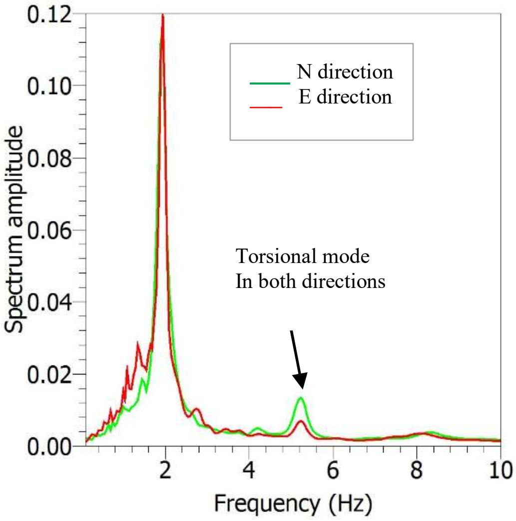

Fig. 3: FRF curves of record points 1

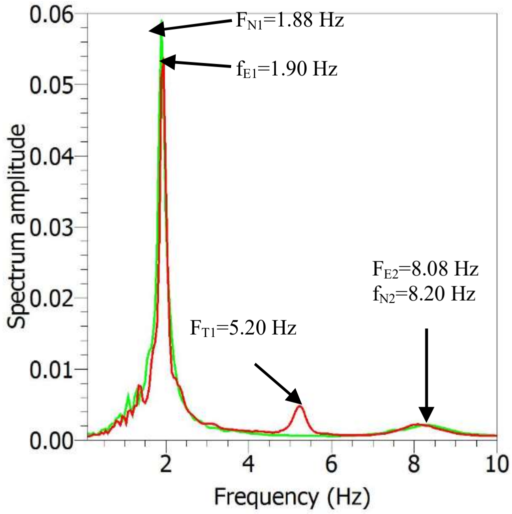

Fig. 4: FRF curves of record points 2.

C. *Identification of the modal damping ratios*

For each identified frequency, a damping ratio has been determined using the random decrement technique (Table 1). Figure 7 shows typical free decay fitting using GEOPSY software [25] where f is the natural frequency, z is the damping ratio and N is the number of windows used in the calculation. The values of the damping ratios are low compared to ratios used in analysis for such type of structure. These values are common for very low excitations (ambient vibrations).

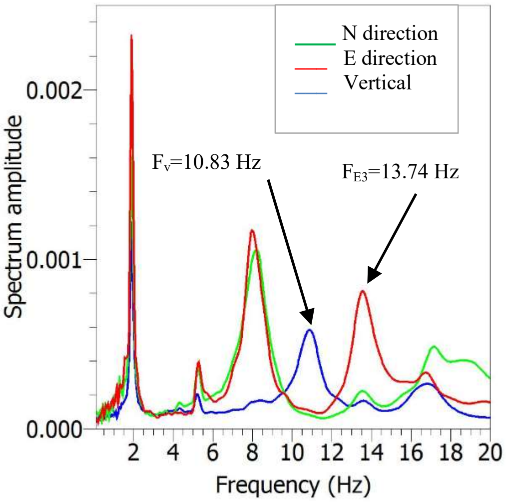

Fig. 5: FRF curves of record points 7.

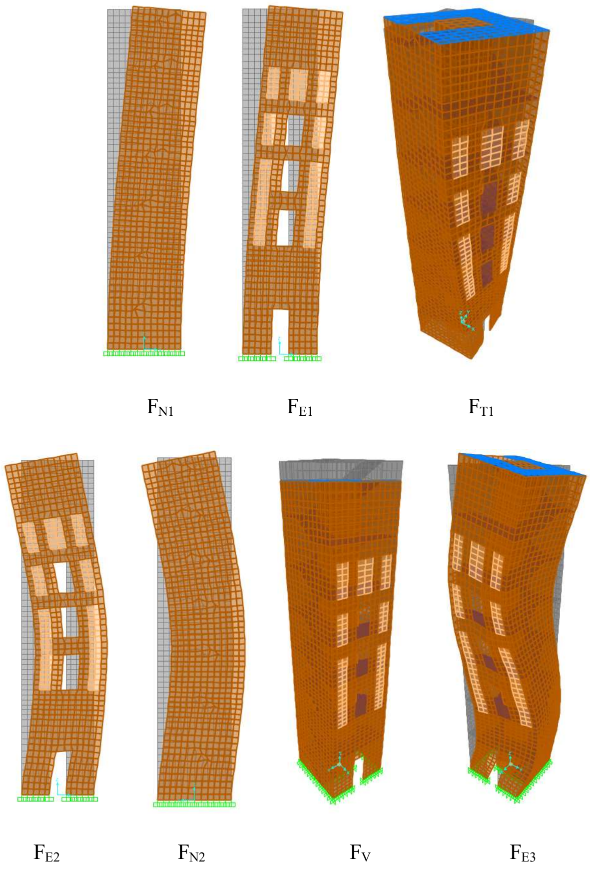

Fig. 6: Numerical mode shapes.

F_{Ei}, F_{Ni}, FV and F_{Ti} denote the ith frequency in the east, north, vertical direction and the torsional mode respectively.

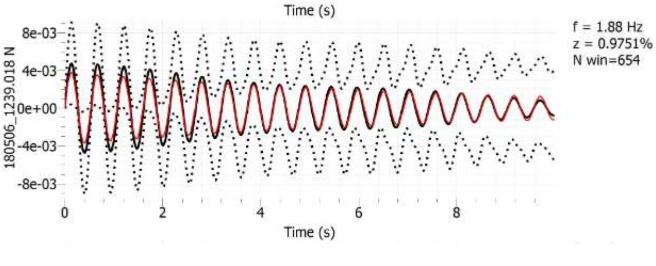

Fig 7: Typical free decay fitting of the fundamental frequency.

IV. EXPERIMENTAL AND NUMERICAL MODAL CORRELATION

A finite element model updating is performed in the frequency domain in two stages. First, the uncertain data of the numerical model is defined. In this instance, the geometry of the structure is simple; it has been precisely defined using the 3-D scan and introduced with high accuracy into the FE model. The mass distribution is also well known; however, the material properties of the old non-homogeneous masonry are uncertain. Therefore, as an inverse problem, the elastic modulus is used as a variable parameter to be identified and the fundamental frequency as an objective function to update iteratively the numerical model with the experimental results. Several iterations are made for changing values of the elastic modulus for which the modal analysis is carried out in each iteration. The best fit for the fundamental frequency is obtained for E = 2.88 GPa. This value is in the range of the elastic modulus of historic stone masonry as reported in many experimental investigations [3, 16, 23]. The numerical frequencies of the model are compared to those determined experimentally in Table1. The fundamental frequencies match closely and remain within 5 % error for second higher modes. The largest error of 17% corresponds to the torsional mode.

Table. I
EXPERIMENTAL AND NUMERICAL FREQUENCIES

No	Direction	Measured (HZ)	Computed (Hz)	Error (%)
Mode 1	Lateral X	1.88	1.89	0.5
Mode 2	Lateral Y	1.90	1.90	0
Mode 3	Torsional	5.20	6.28	17.2
Mode 4	Lateral X	8.08	8.24	2.0
Mode 5	Lateral Y	8.20	8.56	4.2
Mode 6	Vertical	10.83	11.42	5.2
Mode 7	Lateral Y	13.74	16.18	15.1

In a second phase, a frequency correlation function is used. The variable parameter is the modal damping ratios. The numerical FRF is obtained by subjecting the numerical model to a signal with a uniform band limited spectrum. The damping ratios determined using the random decrement technique from the ambient vibration records have been first introduced in the numerical model to compute the initial numerical FRF curve. Then the FRF curve is normalized with respect to the experimental amplitude of the first peak related to the fundamental frequency. The updating procedure is carried out iteratively by adjusting the damping ratios to minimize the FRF error expressed by:

$$\varepsilon H_{ij} \frac{|(H_E)_{ij}-(H_N)_{ij}|}{|(H_E)_{ij}|} \times 100 \qquad (1)$$

Where:

$(H_E)_{ij}$: ith value of the measured FRF curve at point j

$(H_N)_{ij}$: ith value of the numerical FRF curve at point j

The closeness of the measured and numerically calculated FRF is assessed using the Frequency Domain Modal Assurance Criteria (FDAC) technique applied to the FRF as given by the following criterion:

$$FDAC(\omega_E,\omega_N,j)=\frac{\left(\{H_E(\omega_E)\}_j^T\{H_N(\omega_N)\}_j\right)^2}{\left(\{H_E(\omega_E)\}_j^T\{H_E(\omega_E)\}_j\right)\left(\{H_N(\omega_N)\}_j^T\{H_N(\omega_N)\}_j\right)} \qquad (2)$$

Where:

$\{H_E(\omega_E)\}_j$ is the measured FRF curve (column or vector) at point j

$\{H_N(\omega_N)\}_j$ is the calculated FRF curve (column or vector) at point j

ω_E and ω_N correspond to the frequencies at which the FRF amplitudes H_E and H_N are measured and calculated respectively.

Values of the FDAC vary between 0 and 1. When FDAC = 1, it means a perfect matching whereas 0 indicate no correlation at all. After few iterations on the FRF curve of point 2 along the N axis, the best fit is obtained for a damping value $\xi_1 =$ 1.0 % for the fundamental frequency at 1.88 Hz and $\xi_2 =$ 6.0 % for the second mode at 8.20 Hz. The FRF error corresponding to the best fit is εH_{ij} = 0.23 and the FDAC = 0.96. Fig. 8 shows the experimental and the matched numerical FRF curves.

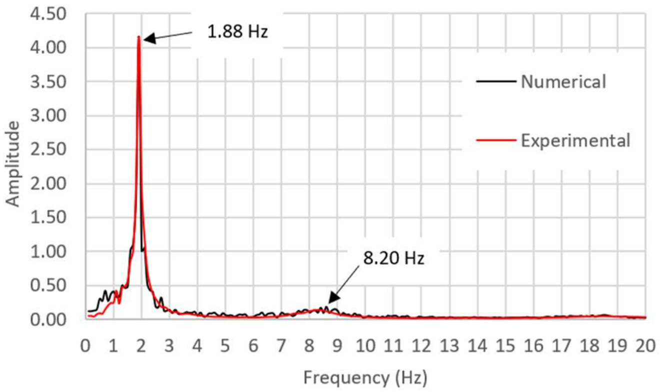

Fig. 8: Experimental and numerical (matched) FRF curve at point 2 along the N axis.

V. SEISMIC PERFORMANCE UNDER EARTHQUAKE GROUND MOTIONS

The seismicity of the region of M'sila, where the site of Kalaa Beni-Hammad is implemented (Maadid), is considered as moderate. It is classified as zone IIa by the Algerian seismic code RPA99v2003 [20]. The CRAAG database [12] reports five seismic events in the region of Msila in the period from1885 to 1965. The last three events of 1946, 1960 and 1965 have similar intensity of VIII and a magnitude of 5.5. The largest most recent earthquake of magnitude 5.2 occurred on 14-05-2010 in the region of Béni-Ilmane at about 70 km from the Kalaa Beni-Hammad site. An elastic spectrum for the site is derived in accordance with the RPA99v2003. This is used to adjust a set of real earthquake records and generate two artificial acceleration ground motions. A set of six acceleration time histories are defined and are used for linear dynamic analyses. The mean spectrum of all ground motions is plotted against the code elastic spectrum in Fig. 9.

Linear time history analyses are carried out to determine the lateral drift and the most stressed zones in the structure. It is worth noting that the results from the linear analysis are limited to portray the stress distribution at an early elastic stage.

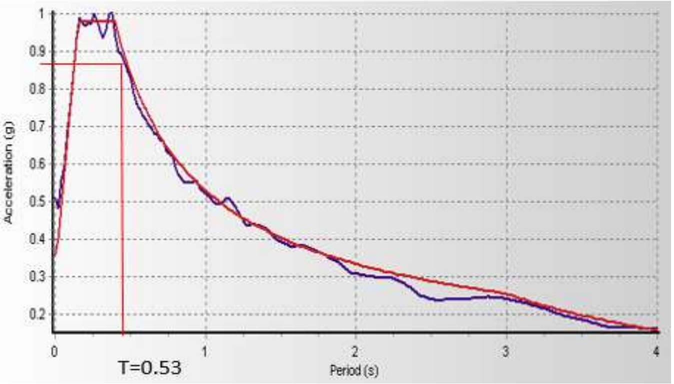

Fig. 9: Code elastic spectrum and mean spectrum of the normalized acceleration ground motions.

The maximum lateral drift at the top, which reaches 9.19 cm (Fig. 10), is very low compared to 1% of the height (25 cm). This is common for this very type of stiff structures with thick exterior walls and a massive masonry core.

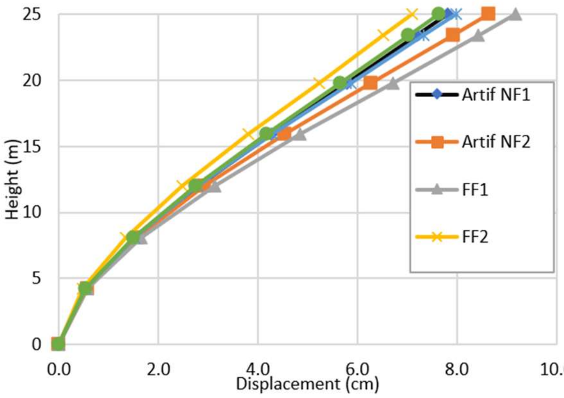

Fig. 10: Lateral drift caused by different acceleration ground motions.

The tensile overstressed zone outspread from the bottom to mid-height of the tower. The shear-overstressed zones, however, are mainly observed near lintels of some openings as shown in Fig. 11 and 12.

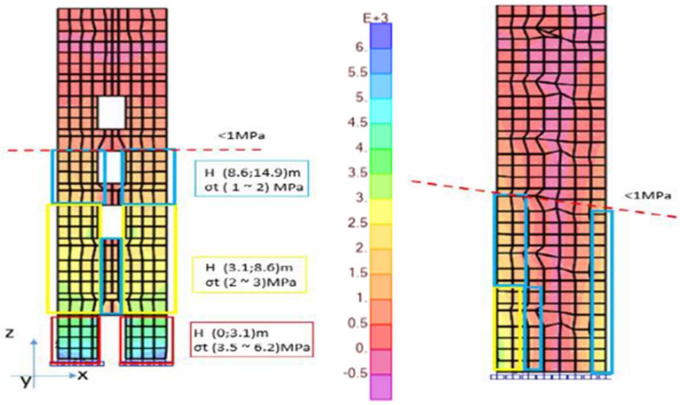

Fig. 11: Tensile stress distribution and locations of overstressed zones.

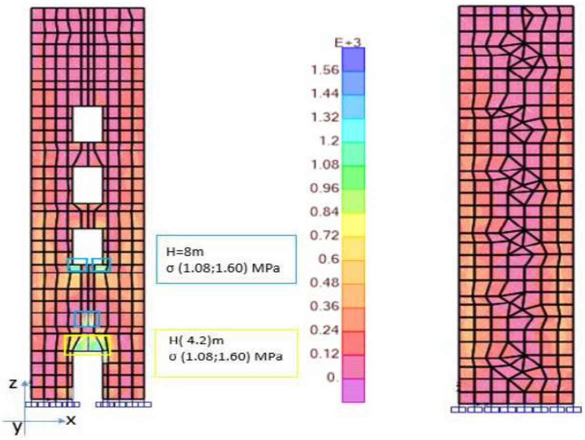

Fig. 12: Shear stress distribution and locations of overstressed zones.

VI. CONCEPTUAL RETROFITTING PROPOSAL

Any strengthening scheme ought to be based on thorough assessment of the edifice, which is out of the scope of the present research work. Nevertheless, a feasibility study of a strengthening solution is proposed. First, to relief the local shear overstressing in some locations of the lower openings, steel or reinforced concrete framing can be incrusted around the openings. This solution has not been developed in this article as the latter is investigating only the steel tie system.

In order to absorb the tensile stresses that would develop in several locations in the ruble stone masonry of the structure as shown above, a lightly post-tensioned steel tie system (Fig. 13) is proposed to bind horizontally and vertically the structure. This is done in such a way that any overstress that may happen in the masonry will be transferred passively to the steel rods. For the purpose of the present study, a simple model based on predefined failure mechanism is used to design the elements of the steel tie system. As shown on Fig. 14, the walls are supposed to be completely maintained by the steel rods and a spectrum analysis is used to determine the tensions in the rods. It has been found that a diameter of 22 mm for all the rods is sufficient to withstand the resulting tensions. Such a system was successfully employed to strengthen an ancient masonry building [8].

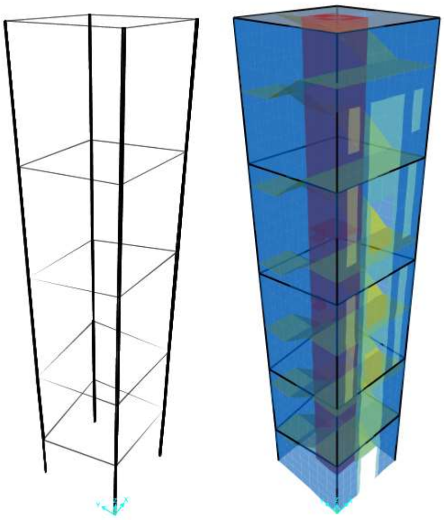

Fig. 13: Steel tie system position in the minaret model

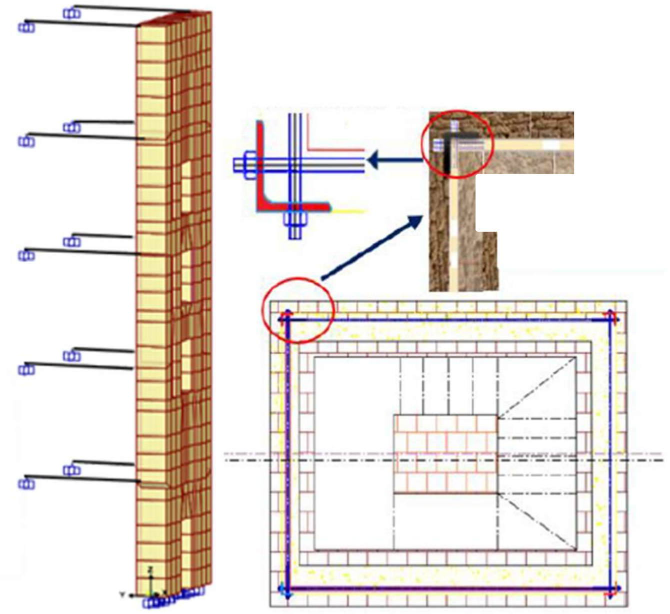

Fig. 14: Strengthening details of the steel tie elements

VII. CONCLUSION

The 24.7 m height minaret of the Kalaa Beni-Hammad is 1000 years old historic monument that has been investigated in this paper in order to assess its structural integrity. An ambient vibration survey has been carried out to update a FE model and identify the material characteristics. For this purpose, a two phases FRF matching procedure has been applied in which the closeness of the measured and numerically calculated FRF is assessed using the Frequency Domain Modal Assurance Criteria with varying modal damping ratios. The results show that the elastic modulus of the material lays within the range of Young’s modulus of historic rubble stone masonry. The updated model is subjected to site spectrum compatible set of acceleration ground motions. Tensile and shear stresses concentration zones are identified and a post-tensioned steel tie system is proposed as a strengthening measure to absorb the tensile stresses that develop in several locations.

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